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Analysis of M/M/c Single and Multi-Server Retrial Queue with Variant Working Vacation Model

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***Abstract*—In this paper, we consider an M /M /c single and multi-server retrial queue with working vacation model. The inter-arrival times are exponentially distributed. Using Markovian chain model, we obtain probability generating function for the number of customers in the orbit. As a consequence, various numerical results are discussed.**

***Index Terms*—M/M/c, Single Server, Multi Server, Working Vacation.**

I. Introduction

Queuing models with vacations have been attracted intense research topics in the queuing theory. Queueing systems with server vacation have been investigated extensively due to their wide applications in several areas including computer communication systems, manufacturing and production systems and inventory systems [1]. Working vacation (WV) is one in which the server provides service at a lower speed during the vacation period rather than stopping service completely [2]. An arriving customer will finds the server free, they get the service immediately. Otherwise, they are forced to enter the orbit of infinite capacity [3]. Requests in the orbit try to get service from the server with a constant retrial rate. During the working vacation period, when the requests depart from the system and no requests being served are in the orbit. Each vacation lasts for a duration that has an exponential distribution. The arriving customers are served with a smaller rate than the normal service rate. At the end of each vacation, the server only takes another new vacation if there is no any new request or any repeated request from the orbit [4]. An arriving customer enters the service facility if the service facility is not full, otherwise the customer joins orbit and repeats its request after random amount of time, called retrial time until the customer gets into the service facility [5].

II. Mathematical Formulation

Let us consider the M /M /c single and multi-server retrial queue with variant working vacations. The parameter λ_B be the inter-arrival times of requests to be exponentially distributed. Request retrials from the orbit of infinite capcity follow a Poisson process with rate ρ. according to Poisson process with rate λ_B, when the server is not on working vacations, the arriving customers to the system. The service rate is μ_B when the system is not on working vacation. The single server takes a working vacation at times when requests being served depart from the system and no requests are in the orbit. Vacation durations are evenly distributed with parameter θ. During the vacation periods arriving customers are served with rate $\mu_r < \mu_b$. At the end of each vacation, the server only takes another new vacation if there is no any new request or repeated request from the orbit.

The system at any time *t* for random variables:

S(*t*) denotes the phase of the system (state of server) at time *t*, while *N*(*t*) represents the the level of the system (number of customers) in the orbit at time *t*.

The single server is constructed as follows:

1) At time $S(t) = 0$, the server is on a working vacation with time *t* and the server does not occupied.
2) At time $S(t) = 1$, the server is on a working vacation at time *t* and the server is busy.
3) At time $S(t) = 2$, The server is not on a working vacation at time *t* and the server is not occupied.
4) At time $S(t) = 3$, the server is not on a working vacation at time *t* and the server is busy.

A. Steady State Distribution

The Markov process

$M(t) = \{S(t), N(t)\}$ on steady state space $Y = \{(x, y) : 0 \le x \le 3, y \ge 0\}$.

The limiting steady state probabilities can be denoted by

$$P(t) = \lim_{t\to\infty} P\,(S(t) = i, N(t) = j$$

Let $\gamma = [\pi_{0,y}, \pi_{1,y}, \pi_{2,y}, \pi_{3,y}]$. Note that $\pi_{2,0} = 0$. (i.e), when there is no customer in the orbit, the probability that the server is not on a working vacation and does not serve a customer is zero.

The steady state equations of the possible transitions rates between the states of the Markov chain Y are identified:

1) The phase transition rate (x, y) to state (z, y) $(\gamma\,(x, y) \in S$ and $(z,y) \in S) \to A_y(x, z)$;
2) The upward transition rate (x, y) to state $(z, y + 1)$ $(\gamma\,(x, y) \in S$ and $(z, y + 1) \in S)$ is represented by $B_y(x, z)$;

3) The downward transition rate from state (x, y) to state $(z, y-1)$ ($\gamma(x, y) \in S$ and $(z, y-1) \in S$) is $C_y(x, z)$.

For N (t) = $y \geq 1$ the following events can happen in the system: Four possible ways by the arrival of a new customer,

1) When the server is not occupied, they changes into busy state (i.e.: $x(t)$ changes either from 0 to 1, or 2 to 3). Since y (t) remains same.
2) When the server is occupied (y (t) = 1 or y (t) = 3), while the customer goes into the orbit.
3) Further, the server becomes free, the customer be departure, (S (t) changes either from 1 to 0 to 3 to 2) and y (t) remains same.
4) Finally, the end of the vacation, then x (t) changes either from 0 to 2 to 1 to 3.

The arrival of a customer from the orbit, then x (t) changes from 0 to 1 to 2 to 3.

Let A*, B* and C* be matrices with the elements A* (x, y), B* (x, z) and C* (x, z), respectively.

Then the matrices can be written as

$$A^* = A = \begin{bmatrix} 0 & \lambda_B & \theta & 0 \\ \mu_r & 0 & 0 & \theta \\ 0 & 0 & 0 & \lambda_B \\ 0 & 0 & \mu_b & 0 \end{bmatrix}$$

$$B^* = B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_B \end{bmatrix}$$

$$C^* = C = \begin{bmatrix} 0 & \rho & 0 & 0 \\ 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & \rho \\ 0 & 0 & 0 & \lambda_B \end{bmatrix}$$

At $N(t) = 0$, when there is no customer in the orbit, the arrival of a new customer may following reasons

When the server is not ocupied, then the server changes to the busy state (i.e.: $S(t)$ changes either from 0 to 1), while $N(t)$ remains unchanged.

When the server is occupied, ($S(t)$ = 1 or 3), then the customer goes into the orbit.

After departure of a customer, the server becomes free ($S(t)$ changes either from 1 to 0 or from 3 to 0) and $N(t)$ remains unchanged.

The status change of the server (i.e.: the end of the vacation), then $S(t)$ changes from 1 to 3.

The generator matrices as shown below:

$$A^*_0 = \begin{bmatrix} 0 & \lambda_B & \theta & 0 \\ \mu_v & 0 & 0 & \theta \\ 0 & 0 & 0 & \lambda_B \\ 0 & 0 & \mu_b & 0 \end{bmatrix}$$

$$B^*_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_B \end{bmatrix}$$

The M/M/c retrial queue at any time t is denoted by

$S^*(t)$ = 0 the server is free at time t

1 the server is busy at time t

$N^*(t)$ be the number of customers in the orbit at time t.

The M/M/c retrial queue is a continuous time discrete state Markov process,

$\{S^*(t), N^*(t)\}$, on the state space $\{(x, y) : x = 0,1, y \geq 0\}$.

The transition rates of the matrices can be written as

$$A^*_y = A^* = \begin{bmatrix} 0 & \lambda_B \\ \mu_1 & 0 \end{bmatrix}$$

$$B^*_y = B^* = \begin{bmatrix} 0 & 0 \\ 0 & \lambda_B \end{bmatrix}$$

$$C^*_q = C^* = \begin{bmatrix} 0 & \rho \\ 0 & 0 \end{bmatrix} \gamma_q \geq 1$$

The polynomial of the retrial M/M/c queue matrix is obtained as

$$Q^*(x) = \begin{bmatrix} Q_{22}x & (\lambda_B x)\rho x^2 \\ \mu_b x & Q_{33}x \end{bmatrix}$$

$$\varphi_j = \sum_{1=3}^{4} \alpha^*_i x^*_i \Psi^*_i$$

The eigen values namely x_3, x_4 and x_5, which form the subset of the eigen values are associated with the retrial M/M/c queue with working vacations.

The eigen vector x_3 of

$Q^*(x)$ is $\Psi^*_3 = [1, 0]$,

while

$\Psi^*_4 = [\mu_V, \rho + \lambda_B]$ is the corresponds to eigenvectors of eigen value x_4.

where

α^*_i is the coefficients, which can be determined from the balance equation for $N^*(t) = 0$ and the normalization equation

$$\alpha_3^* = \frac{\alpha\mu_b - \alpha\lambda - \lambda^2}{(\alpha+\lambda)\mu_b}$$

$$\alpha_4^* = \frac{\alpha\lambda\mu_b - \alpha\lambda - \lambda^2}{(\alpha+\lambda)\mu_b^2}$$

The probability generating function of the number of customers in the orbit, given the server is busy, is

$$N(z) = \sum_{j=0}^{\infty} x_4^j z^j \alpha_4^*(\alpha+\lambda) = \frac{(\alpha+\lambda)\alpha_4^*}{1 - z x_4}$$

B. Multi-Server Retrial Queue with Working Vacations

Similarly, the multi-server retrial queue with vacations under various vacation policies can be modeled by Markov process with matrix components. The stationary distribution of the queueing systems with retrials and work vacations on the steady state space is

X = m :

Where,

m = {0 , 1 , 2 , . . .}

The generator form is

$$Q^* = \begin{pmatrix} B_0 & A_0 & 0 & 0 & 0 & \dots \\ C_1 & B_1 & A_1 & 0 & 0 & \dots \\ 0 & C_2 & B_2 & A_2 & 0 & \dots \\ 0 & 0 & C_3 & B_3 & A_3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

The matrices A_n ; B_n and C_n are given by

$$A_m = \begin{pmatrix} 0 & \\ & A_j \end{pmatrix}$$

$$C_m = \begin{pmatrix} 0 & C_0 & & \\ & 0 & \ddots & \\ & & \ddots & C_{j-1} \\ & & & 0 \end{pmatrix}, \ m \geq 1$$

$$B_m = \begin{pmatrix} B_{00} & A_{01} & 0 & 0 & 0 \\ C_{10} & B_{11} & A_{12} & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & B_{j-1,j-2} & B_{j-1,j-1} & B_{j-1,j} \\ \vdots & \vdots & \vdots & B_{j,j-1} & B_{j,j} \end{pmatrix}, \ m \geq 0$$

Let

$N^*(t)$ be the number of customers in orbit and

$S^*(t)$ be the state of the server at time t.

$I_a(t)$ be the phase of the arrival process and $J_s(t)$ be the server state at time t defined by

$I_s(t) = 0$, the servers are available

i, the phase of working vacation time is of *i*, $1 \leq i \leq w$.

The Markov process

$Z^* = \{Z^*(t), t \geq 0\}$ with $Z^*(t)$

$= (N^*(t), S^*(t), I_a(t), I_s(t)$ is a continuous time discrete state Markov chain on the state space

$S^* = \sum_{m=0}^{\infty} m,$

Where,

m = $\{(m, j, i, k) : 0 \leq j \leq J, 1 \leq i \leq l, 0 \leq 6 \leq w\}$; $m \geq 0$.

The matrix components of A_m and C_m of the generator Q of Z^* are given as

$A_m = D \otimes I_{w+1}$

$Bj_{,j+1} = D \otimes I_{w+1}$

$Cj = \gamma_n I_{w+1}$

Where,

$$M_0(\mu)\, p = \begin{pmatrix} 0 & \mu\delta \\ 0 & \mu I_w \end{pmatrix}$$

$$M_1(\mu) = \begin{pmatrix} \mu & 0 \\ 0 & \mu I_w \end{pmatrix}$$

C. Computation of the matrix R

We can obtain the matrix R as a limit of R_n, n = 0 ; 1 ; 2 ; . . ., where

$R^* = 0$, $R_n = (A_0 + R^* A_2)(-A_1)^{-1}$, N = 1,2

It is also recommended for computing R.

D. Numerical Results

First, we depict the behavior of y_0 (K, N)1 = Pr(X_0 = 0) for the convergence of stationary distribution as truncation levels K and N increase.

The parameters are

$\mu = 1{:}0$, s = 10, $\lambda = 10{:}0$, $\theta = 0{:}5$, $\gamma_n = 10n$, (n = 0; 1 ; 2; . . .) and the probabilities $a_i(j)$, $b_i(j)$, $c_i(j)$ are as follows:

$\alpha_0^* = 0.02$, $\alpha_0 = 0.05$, $\beta_0^* = 0.03$, $\beta_0 = 0.05$, $\gamma_0^* = 0.2$, $\gamma_0 = 0.25$,

$\alpha_1^* = 0.28$, $\alpha_1 = 0.35$, $\beta_1^* = 0.37$, $\beta_1 = 0.5$, $\gamma_1^* = 0.8$, $\gamma_1 = 0.75$,

$\alpha_2^* = 0.7$, $\alpha_2 = 0.6$, $\beta_2^* = 0.6$, $\beta_2 = 0.45$.

The values K, y_0 (K,N)1 approaches to a constant as N increases and for large N, y_0 (K,N)1 decreases monotonically as K increases. Thus we can see that y_0 (K,N)1 converges to a constant.

Now we illustrate the algorithmic for stationary distribution were investigate to performance the characteristics. Let N_0 and N_1 be the number of customers in orbit and service requests, respectively in stationary state and $S_0 = E[X_0]$, $L_1 = E[X_1]$. The probability P_r and the loss probability P_L are given by the formulae

$P_r = P(X_1 \geq s) = x_{ij}$,

$P_l = \frac{1}{\lambda b}\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \lambda a_0(j) + \gamma_i b_0(j) + \theta_j c_0(j) x_{ij}\right)$

III. Conclusion

In this paper, we consider both single and multi-server retrial queuing system with working vacations. When the orbit becomes empty at the time of service completion for a positive customer, the server goes for a working vacation. The server works at a lower service rate during working vacation (WV) period. If there are customers in the system at the end of each vacation, the server becomes idle and ready for serving new arrivals with probability p (single WV) or it remains on vacation with probability q (multiple WVs). By using the supplementary variable technique, we found out the steady state probability generating function for the system and its orbit. System performance measures, reliability measures and stochastic decomposition law are discussed. Finally, some numerical examples and cost optimization analysis are presented.

References

[1] M. Jain a, D. Chauhan, *International Journal of Engineering*, Working Vacation Queue with Second Optional Service and Unreliable Server, 25, 223-230, 2012.

[2] V.M. Chandrasekaran, K. Indhira, M.C. Saravanarajan, P. Rajadurai, International Journal of Pure and Applied Mathematics, *A survey on working vacation queueing models*, 106, 6, 33-41, 2016.

[3] S. R. Chakravarthya, A. Krishnamoorthy, V.C. Joshua, *Performance Evaluation*, Analysis of a multi-server retrial queue with search of customers from the orbit, 63, 776–798, 2006.

[4] T. V. Do, Acta Informatica, M/M/1 retrial queue with working vacations, 47, 67–75, 2010.

[5] Y. W. Shin, Applied Mathematics and Computation, Algorithmic approach to Markovian multi-server retrial queues with vacations, 250, 287–297, 2015.