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Odd and Even Formation for Graceful Labeling

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***Abstract*—Over the past decades, graph theory has established itself as a worthwhile mathematical discipline and there are many application of graph theory to a wide variety of subjects. Labeled graph have several type of application in secret writing theory. Tagged graph plays vital role within the study of X-ray physics, communication network and to see best circuit layouts. Labeling of a graph is an assignment of labels (numbers) to its edges that satisfy some conditions. Graph labeling has become a vigorous space in graph theory [1-6].**

***Index Terms*—Graceful Labeling, Vertex, Cycle, Edge**

I. Introduction

The graph theory is one in all the sector of separate arithmetic that cuts across wide rages of disciplines of human understanding not solely within the space of maths. The later a part of last century has witnessed intense activity in graph theory. Development of engineering science intensify the analysis add the sector. There are several fascinating field of analysis in graph theory. A number of them square measure domination in graph, coloring graph, topological graph theory, fuzzy graph theory ad labeling of separate structures [1-6].

Definition: 1.1
If the vertices area unit appointed values subject to sure conditions then it's called graph labeling.

Definition: 1.2
A function f is called graceful labeling of a graph G if f (V) $V \to \{0,1,\ldots q\}$ is injuctive and the inducted function f*: $E \to \{0,1,\ldots,q\}$ defined as $f(e = uv) = |f(u) - f(v)|$ be the bijective. A graph which admits graceful labeling is called a graceful graph.

II. Mathematical Formulation

Theorem:
Duplication of a discretional vertex of C_m which produces a sleek graph.
Proof:
Let $v_1, v_2, v_3 \ldots, v_m$ be the vertices of the cycle C_m and G be the graph obtained by duplicating an arbitrary vertex of C_m. Without loss of generality let this vertex be v_1 the newly added vertex be v^*_1. To define f (V) $\to \{0,1,2,3 \ldots, q\}$ following four cases are to be considered.
Case (i)
When $m \equiv 0 (\text{mod } 4)$; $m \cong 4$
$f(v^*_1) = 0$
$f(v_1) = \frac{m}{2} + 1$
$f(v_i) = (m + 2) - (\frac{i-2}{2})$; when i is even and for $2 \le i \le \frac{m}{2} + 2$
$= \frac{i-1}{2}$; when i is odd and for $2 \le i \le \frac{m}{2} + 2$
$f(v_i) = (m + 2) - (\frac{i-1}{2})$; when i is odd and for $\frac{m}{2} + 3 \le i \le m$
$= \frac{i-2}{2}$; when i is even and for $\frac{m}{2} + 3 \le i \le m$
If m = 4 is to be dealt separately.
Case (ii)
When $m \equiv 1 (\text{mod } 4)$
$f(v'_1) = 0$
$f(v_1) = \frac{m+1}{2}$
$f(v_i) = (m + 2) - (\frac{i-2}{2})$;
when i is even and for $2 \le i \le \frac{m+1}{2}$
$= \frac{i-1}{2}$
when i is odd and for $2 \le i \le \frac{m+1}{2}$
$f(v_i) = \frac{i}{2}$;
when i is even and for $\frac{m+3}{2} \le i \le m$
$= (m + 2) - (\frac{i-2}{2})$;
when i is odd and for $\frac{m+3}{2} \le i \le m$
Case (iii)
When $m \equiv 2 (\text{mod } 4)$**;** $m \neq 6$
$f(v^*_1) = 0$
$f(v_1) = \frac{m}{2} + 3$
$f(v_i) = (m + 2) - (\frac{i-2}{2})$;
when i is even and for $2 \le i \le \frac{m+4}{2}$
$= \frac{i-1}{2}$;
when i is odd and for $2 \le i \le \frac{m+4}{2}$
$f(v_i) = \frac{i}{2}$; for $i = \frac{m+4}{2} + 1$
$f(v_i) = (m + 2) - (\frac{i-3}{2})$;
when i is odd and for $\frac{m+4}{2} \le i \le m$
$= \frac{i+2}{2}$;
when i is even and for $\frac{m+4}{2} \le i \le m$
For n = 6; the corresponding graph and its graceful labeling is shown.

When m ≡ 3(mod 4)

$f(v^*_1)=0$

$f(v_1) = \frac{m+1}{2}$

$f(v_i) = (m + 2) - (\frac{i-2}{2});$

when i is even and for $2 \leq I \leq \frac{m+1}{2}$

$= \frac{i-1}{2};$ when i is odd.

$f(v_i) = (m + 2) - (\frac{i-1}{2});$ when i is odd and for $\frac{m+3}{2} \leq i \leq m$

$= \frac{i}{2};$ when i is even and for $\frac{m+3}{2} \leq i \leq m$

The graceful labeled ascribed by the vertex in cycle C_m

Illustration:

The graph obtained by duplicating the vertex v_1 of cycle C_8 is shown in Figure. (For case-1: m ≡ 0(mod 4)) and $e_f(0) = 8$ and $e_f(1) = 8$.

Theorem:

An Edge of cycle of Even Order is in Graceful labeling

Proof:

Let us assume that $v_1, v_2 \ldots v_m$ are vertices of the cycle C_m, where *m* is even G be the graph on edge of C_m.

Without loss of generality assume that $e_f = v^*_1 v^*_2$ be the newly added edge to duplicate the edge $e_f = v_1v_2$ in C_m.

To define f (V)→{0, 1,2,3 . . . , *q*} following two cases are to be considered.

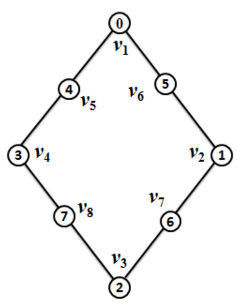

Fig. 1. Duplication of a vertex in c_8 and its graceful labeling

Case (i)

m ≡ 0(mod 4); m≠4, m≠ 8

$f(v^*_1) = \frac{m}{2} + 4$

$f(v^*_2) = \frac{m}{2}$

$f(v_i) = (m + 3) - \frac{i-1}{2};$ when i is odd and for $1 \leq i \leq \frac{m}{2} + 2$

$= \frac{i-2}{2};$ when i is even and for $1 \leq i \leq \frac{m}{2} + 2$

$f(v_i) = \frac{i-1}{2};$ for $i = \frac{m}{2} + 3$

$f(v_i) = (n + 3) - \frac{i}{2};$ when i is even and for $\frac{m}{2} + 4 \leq i \leq m - 1$

$= \frac{i-1}{2};$ when i is odd and for $\frac{m}{2} + 4 \leq i \leq m - 1$

$f(v_m) = \frac{m}{2} + 2$

The labeling for the graphs corresponding to C_4 and C_8 are to be dealt separately.

Case (ii)

When m ≡ 2 (mod 4)

$f(v^*_1) = \frac{m}{2} - 1$

$f(v^*_2) = \frac{m}{2}$

$f(v_i) = (m + 3) - \frac{i-1}{2};$ when i is odd and for $1 \leq i \leq \frac{m}{2} + 2$

$= \frac{i-2}{2};$ when i is even and for $1 \leq i \leq \frac{m}{2} + 2$

$f(v_i) = (m + 3) - \frac{i+2}{2};$ when i is even and for $\frac{m}{2} + 3 \leq i \leq m$

$= \frac{i-3}{2};$ when i is odd and for $\frac{m}{2} + 3 \leq i \leq m$

Therefore, the function f provides graceful labeling for the graph obtained by the duplication of an edge in even cycle C_m.

III. CONCLUSION

The graceful labeling on the vertex and edge through the odd or even or both of the graph G are discussed. From the point of view the even term of graceful label assigned on the vertices and on the edge C_m.

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