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Novel Approach to Solve the Assignment Problem

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***Abstract*: Herein, we solve an assignment problem. It was observed that the solution is the minimum achievable result. The novel approach can be applicable for various kind of Assignment problem. It may benefit from the proposed approach for supply and demand. We get the optimal solution which is less than optimal solutions or other three methods.**

***Keywords*: Assignment problem, Transportation, Hungarian.**

1. Introduction

Transportation model usually involves several modes of transports and services from various supply origin to multiple demand purpose within the given constraint of supply and demand in such a way that the total transportation cost is minimized.

The assignment models are looks as like as Transportation Problem. The best person for the job is an apt description of what the assignment model seeks to accomplish.

The situation can be carried out by the workers to assigning the jobs. However, some worker could take any job albeit with varying degrees of skill.

A job that happens to match a worker's skill costs less than that in which the operator is not as skillful.

The objective of the program is to analyze the assigning the jobs to workers.

Let as consider an assignment model with *m* workers and *m* jobs is shown in the Table.

Let as consider an assignment model with *m* workers and *m* jobs is shown in the Table. The element C_{ij} represent the cost of assigning worker i to job j.

[i, j = 1,2, ……………… m].

There is no loss in generally in assuming that the number of workers always equals the number of jobs, because we can always add fictitious workers (or) fictitious jobs to effect this results.

The supply amount at each source and the demand amount at each destination exactly equals to 1. The cost of "transporting" worker i to j is c_{ij}.

If effect, the assignment model can be solved directly as a regular transportation model. Nevertheless, the fact that all the supply and demand amounts equal 1 has led to the development of a simple solution algorithm called Hungarian Method.

2. Mathematic Formulation

A. Assignment Problem

Consider an assignment problem of assigning n jobs to n machines (one job to one machine).

Let c_{ij} be the unit cost of assigning i^{th} machine to the j^{th} job and

$$\text{Let } x_{ij} = \begin{cases} 1. \text{ if } j^{th} \text{ job is assigned to } i^{th} \text{ machine} \\ 0. \text{ if } j^{th} \text{ job is not assigned to } i^{th} \text{ machine} \end{cases}$$

The assignment model is then given by the following LLP

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{m} x_{ij} = 1, j = 1,2, \dots m$$

$$\sum_{j=1}^{m} x_{ij} = 1, i = 1,2, \dots m$$

and x_{ij} = 0 (or) 1.

B. Assignment Algorithm (or) Hungarian Method

If the number of rows and columns of an assignment problem are equal, then the given problem is balanced. Then proceed to,

Step 1: If the number of rows and columns are not equal, then it should be balanced before applying the algorithm.

Step 2: For the initial value matrix, establish every row's minimum and take off it from all the entries of the row.

Step 3: For the matrix ensuing from step one, establish every column's minimum and take off it from all entries of the column.

Step 4: Identify the optimal assignment as the one associated with zero element of the matrix obtained in step 2.

Note 1: In case some rows or columns contain more than one zero, encircle any unmarked zero randomly and cross all other zeros in its column or row. Continue this process until no zero is left unmarked or encircled.

Note 2: If the given assignment problem is maximum, convert it in to a minimization assignment problem by max Z = - min (-Z) and multiply all the given cost elements by -1 in the

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cost matrix and then solve by assignment algorithm.

3. Methods for Solving Assignment Problem

A. Numerical Example

Here, the processing times in hours for the jobs when assigned to the different machines are given below. Assigning the machines for the jobs so that we calculate the overall processing time is minimum.

Machines						
		M_1	M_2	M_3	M_4	M_5
Jobs	J_1	9	22	58	11	19
	J_2	43	78	72	50	63
	J_3	41	28	91	37	45
	J_4	74	42	27	49	39
	J_5	36	11	57	22	25

Solution: The cost matrix of the given problem is

9	22	58	11	19
43	78	72	50	63
41	28	91	37	45
74	42	27	49	39
36	11	57	22	25

Hence, the number of rows is equal to the number of columns in the cost matrix, the given assignment problem is balance.

Step 1: Select the smallest cost element in each row and remove this from all the elements of the relative row, we get the reduced matrix:

0	13	49	2	10
0	35	29	7	20
13	0	63	9	17
47	15	0	22	12
25	0	46	11	14

Step 2: Select the smallest cost element in each column and remove this from all the elements of the relative column, we get the following reduced matrix:

0	13	49	0	0
0	35	29	5	10
13	0	63	7	7
47	15	0	20	2
25	0	46	9	4

Step 3: Now we shall identify the rows successively. Second row carry a single unmarked zero, encircle this zero and cross all other zeros in its column. Third row carry a single unmarked zero, encircle this zero and cross all other zeros in its column. Fourth row carry a single unmarked zero, encircle this zero and cross all other zeros in its column. After this all the rows are having more than one unmarked zero, so go for columns.

Identify the columns successively, fourth column contains exactly one unmarked zero, encircle this zero and cross all other zeros in its row. After identifying all the rows and columns, we get,

~~0~~	13	49	(0)	~~0~~
(0)	35	29	5	10
13	(0)	63	7	7
47	15	(0)	20	2
25	~~0~~	46	9	4

Step 4: Since the 5th row and column do not have any assignment the present assignment is not optimal.

Step 5: Draw the minimum attainable range of horizontal and vertical lines therefore on cover all the zeros.

0	13	49	0	0	
0	35	29	5	10	
13	0	63	7	7	√
47	15	0	20	2	
25	0	46	9	4	√
	√				

Step 6: In this 4 is the smallest element not covered by these straight lines. Subtract 4 from all the not covered elements and add 4 to all those elements which are placing in the intersection of these straight lines and do not change the remaining elements which lie on these straight lines, we get the succeeding matrix.

0	17	49	0	0
0	39	29	5	10
9	0	59	3	3
47	19	0	20	2
21	0	42	5	0

Since each row and each column contains not less than one zero, we investigate the rows and columns successively, i.e., repeat step 3 above, we get,

~~0~~	17	49	(0)	0
(0)	39	29	5	10
9	(0)	59	3	3
47	19	(0)	20	2
21	~~0~~	42	5	(0)

In the above matrix, each row and each column contains absolutely assignment (i.e., exactly one bounded zero), therefore the current assignment is optimal.

Therefore, the optimum assignment schedule is $J_1 \rightarrow M_4$, $J_2 \rightarrow M_1$, $J_3 \rightarrow M_2$, $J_4 \rightarrow M_3$, $J_5 \rightarrow M_5$ and the optimum (minimum)

processing time
= 11 + 43 + 28 + 27 + 25 hrs
= 134 hrs.

4. Conclusion

Herein, we used to solve an assignment problem in an easy way. The solution gives the minimum achievable result. The use of assignment method gives a well ordered and clear solution. Hence, this method gives a novel approach which is easy to solve assignment problem.

References

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