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Numerical Analysis of Fracture of Unfired PV: A Review

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***Abstract*: A pressure vessel is a container designed to hold gases or liquids at pressure substantially different from the pressure. Designing thin-wall pressure vessels to store fluids is a common practice in engineering. By definition, a thin-wall pressure vessel requires that the plate thickness be small as compared with the vessel's internal diameter t<<d If curved plates are welded to make pressure vessels, the welded joints become the weakest areas of the structure since weld defects can be the source of cracks during service. The design assessment use criteria of leak before break and criteria for crack stability. These two criteria signify the fracture of pressure will occur or not. The material considered are steel 4340,4335 and 350 Maraging.**

***Keywords*: Crack, Fracture, Leak before break (LBB), Pressure vessel.**

1. Introduction

A pressure vessel is a container designed to hold gases or liquids at pressure substantially different from the pressure. Pressure vessels can be dangerous, and fatal accidents have occurred in the history of their development and operation. Consequently, pressure vessel design, manufacture, and operation are regulated by engineering authorities backed by legislation. For these reasons, the definition of a pressure vessel varies from country to country. In most countries, vessels over a certain size and pressure must be built to a formal code. In the United States that code is the ASME Boiler and Pressure Vessel Code (BPVC). In Europe the code is the Pressure Equipment Directive. Information on this page is mostly valid in ASME only. These vessels also require an authorized inspector to sign off on every new vessel constructed and each vessel has a nameplate with pertinent information about the vessel, such as maximum allowable working pressure, maximum temperature, minimum design metal temperature, what company manufactured it, the date, its registration number (through the National Board), and ASME's official stamp for pressure vessels (U-stamp). The nameplate makes the vessel traceable and officially a ASME Code vessel [1].

2. Literature review

Fracture mechanics refers to the mechanics of solids containing planes of displacement discontinuities (cracks) with special attention to their growth. Fracture mechanics is a failure theory that Determines material failure by energy criteria, possibly in conjunction with strength (or yield) criteria Considers failure to be propagating throughout the structure rather than simultaneous throughout the entire failure zone or surface. It is a useful method of determining Stress and flaw size Fracture toughness Fatigue crack growth Stress corrosion crack growth behaviour Fracture mechanics has been used heavily in the aerospace, and ship industries with a recent extension to the ground vehicle industry. Fracture mechanics is the field of mechanics concerned with the study of the propagation of cracks in materials. It uses methods of analytical solid mechanics to calculate the driving force on a crack and those of experimental solid mechanics to characterize the material's resistance to fracture. Fracture mechanics is an important tool in improving the mechanical performance of components. It applies to the microscopic crystallographic defects found in materials in order to predict the macroscopic mechanical failure of bodies. Fractography is widely used with fracture mechanics to understand the causes of failures and also verify the theoretical failure predictions with real-life failures. Interior and surface flaws arising from the manufacturing process are found in all metal structures. Not all such flaws are unstable under service conditions. Fracture mechanics analyses flaws to determine which are safe and which are liable to propagate as cracks and cause failure of the flawed structure[3] There are two types of fracture mechanics Linear-elastic fracture mechanics - the basic theory of fracture that deals with sharp cracks in elastic bodies Elastic-plastic fracture mechanics - the theory of ductile fracture, usually characterized by stable crack growth (ductile metals) Fracture mechanics can estimate the maximum crack that a material can withstand before it fails, taking into consideration: Fracture mechanics can be used in Material selection and alloy development. Determining the significance of defects Monitoring and control Failure analysis.

A. Modes of Failure of Pressure Vessels

Two basic modes of failure are assumed for the design of pressure vessels. These are: Elastic failure, which is governed by the theory of elasticity; and Plastic failure, which is governed by the theory of plasticity. Except for thick-walled pressure vessels, elastic failure is assumed for the design of pressure vessel. When the material is stretched beyond the elastic limit, excessive plastic deformation or rupture is expected. In a thick-walled pressure vessel, circumferential and radial stresses are initially both maximum on the inner surface. However, failure of the shell does not begin at the bore but in sections on the outer surface of the shell. Although parts on the inner surface reach yield point first, they are incapable of failing because they are restricted by the outer portions of the shell. At a pressure above the elastic-breakdown, the region of plastic flow or “overstrain” moves radially outward and causes the circumferential stress to reduce at the inner layers and to increase at the outer layers resulting to the eventual failure beginning from the outer surface of the vessel where the maximum hoop stress is finally reached. Therefore, plastic failure is assumed for the design of a thick-walled pressure vessel. The two modes of failure are related to the traditional approach of structural design where the anticipated design stress is normally compared to the flow properties of the material, where the material is assumed to be adequate if its strength is greater than the expected applied stress. This implies that, the most commonly used factor in the design of pressure vessels is that of maintaining the induced stresses within the elastic region of the material of construction. This is done in order to avoid excessive plastic deformation or failure of the material when the yield point is exceeded. However, the presence of undetected crack on the wall of a pressure vessel can severely reduce its strength. That is why there have been incidences of failure of pressure vessels that could not be attributed to strength but to brittle and ductile fracture. Therefore, the fracture mechanics approach to the structural design of engineering components such as a pressure vessel must be applied in order to ensure the structural integrity of the component is guaranteed where there is a real possibility of fracture of the component in service.

Three Modes of Cracking-
Mode I -opening mode
Mode II -in-plane shearing/sliding mode
Mode III -out-of-plane shearing/tearing mode

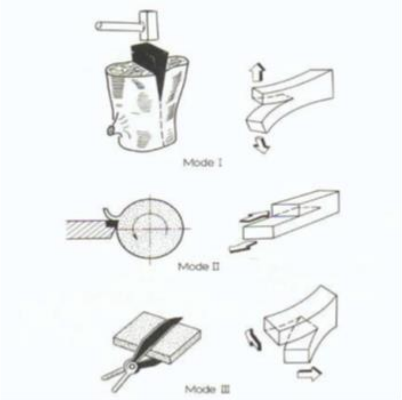

Fig. 1. Modes of fracture

In the mode I or opening mode: The body is loaded under tensile forces, such that the crack surfaces are pulled apart in the opposite direction. The deformations are then symmetric with respect to the planes perpendicular to the y axis and the z axis.

In the mode II or sliding mode: The body is loaded under shear forces applied parallel to the cracked surfaces, which slide over each other in the direction of applied forces. the deformations are then symmetric with respect to the plane perpendicular to the z axis and skew symmetric with respect to the plane perpendicular to the y axis.

In the mode III or tearing mode: The body is loaded under shear forces parallel to the crack front and the crack surfaces slide over each other in the z direction. The deformations are then skew-symmetric with respect to the plane perpendicular to the z and the y axis [4].

B. *Griffith's criterion*

Fracture mechanics was developed during World War I by English aeronautical engineer A. A. Griffith – thus the term Griffith crack – to explain the failure of brittle materials.

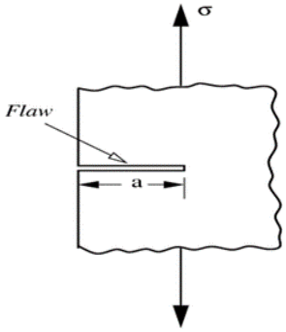

Fig. 2. Griffiths criteria

Griffith's work was motivated by two contradictory facts:

The stress needed to fracture bulk glass is around 100 MPa (15,000 psi). The theoretical stress needed for breaking atomic bonds of glass is approximately 10,000 MPa (1,500,000 psi).

A theory was needed to reconcile these conflicting observations. Also, experiments on glass fibres that Griffith himself conducted suggested that the fracture stress increases as the fibre diameter decreases. Hence the uniaxial tensile strength, which had been used extensively to predict material failure before Griffith, could not be a specimen-independent material property. Griffith suggested that the low fracture strength observed in experiments, as well as the size-dependence of strength, was due to the presence of microscopic flaws in the bulk material.

To verify the flaw hypothesis, Griffith introduced an artificial flaw in his experimental glass specimens. The artificial flaw was in the form of a surface crack which was much larger than other flaws in a specimen. The experiments showed that the product of the square root of the flaw length (a) and the stress at fracture (σ_f) was nearly constant, which is expressed by the equation:

$$\sigma_f\sqrt{a} \approx C$$

An explanation of this relation in terms of linear elasticity theory is problematic. Linear elasticity theory predicts that stress (and hence the strain) at the tip of a sharp flaw in a linear elastic material is infinite. To avoid that problem, Griffith developed a thermodynamic approach to explain the relation that he observed.

The growth of a crack, the extension of the surfaces on either side of the crack, requires an increase in the surface energy. Griffith found an expression for the constant *C* in terms of the surface energy of the crack by solving the elasticity problem of a finite crack in an elastic plate.

Briefly, the approach was:

- Compute the potential energy stored in a perfect specimen under a uniaxial tensile load.
- Fix the boundary so that the applied load does no work and then introduce a crack into the specimen. The crack relaxes the stress and hence reduces the elastic energy near the crack faces. On the other hand, the crack increases the total surface energy of the specimen.
- Compute the change in the free energy (surface energy − elastic energy) as a function of the crack length. Failure occurs when the free energy attains a peak value at a critical crack length, beyond which the free energy decreases as the crack length increases, i.e. by causing fracture. Using this procedure, Griffith found that

$$C = \sqrt{\frac{2E\gamma}{\pi}}$$

where E is the Young's modulus of the material and γ is the surface energy density of the material. Assuming E = 62 GPa and $\gamma = 1\ J/m^2$ gives excellent agreement of Griffith's predicted fracture stress with experimental results for glass.

Griffith's criterion has been used by Johnson, Kendall and Roberts also in application to adhesive contacts. Recently, it was shown that direct application of the Griffith criterion to a single numerical "cell" leads to a very robust formulation of the Boundary Element Method.

For materials highly deformed before crack propagation, the linear elastic fracture mechanics formulation is no longer applicable and an adapted model is necessary to describe the stress and displacement field close to crack tip, such as on fracture of soft materials.

C. Irwin's modification

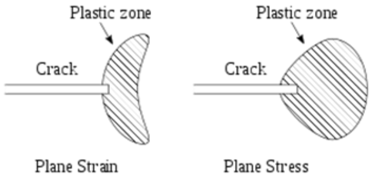

Fig. 3. The plastic zone around a crack tip in a ductile material

Griffith's work was largely ignored by the engineering community until the early 1950s. The reasons for this appear to be,

a) In the actual structural materials, the level of energy needed to cause fracture is orders of magnitude higher than the corresponding surface energy
b) In structural materials there are always some inelastic deformations around the crack front that would make the assumption of linear elastic medium with infinite stresses at the crack tip highly unrealistic.

Griffith's theory provides excellent agreement with experimental data for brittle materials such as glass. For ductile materials such as steel, although the relation still holds, the surface energy γ predicted by Griffith's theory is usually unrealistically high.

$$\sigma_f\sqrt{a} \approx C$$

A group working under G. R. Irwin at the U.S. Naval Research Laboratory (NRL) during World War II realized that plasticity must play a significant role in the fracture of ductile materials.

In ductile materials (and even in materials that appear to be brittle), a plastic zone develops at the tip of the crack. As the applied load increases, the plastic zone increases in size until the crack grows and the elastically strained material behind the crack tip unloads. The plastic loading and unloading cycle near the crack tip leads to the dissipation of energy as heat. Hence, a dissipative term has to be added to the energy balance relation devised by Griffith for brittle materials. In physical terms, additional energy is needed for crack growth in ductile materials as compared to brittle materials.

Irwin's strategy was to partition the energy into two parts:

- The stored elastic strain energy which is released as a crack grows. This is the thermodynamic driving force for fracture.
- The dissipated energy which includes plastic dissipation and the surface energy (and any other dissipative forces that may be at work). The dissipated energy provides the thermodynamic resistance to fracture. Then the total energy is

$$G = 2\gamma + G_p$$

where γ is the surface energy and G_p is the plastic dissipation (and dissipation from other sources) per unit area of crack growth.

The modified version of Griffith's energy criterion can then be written as,

$$\sigma_f\sqrt{a} = \sqrt{\frac{E\,G}{\pi}}.$$

For brittle materials such as glass, the surface energy term dominates

$$G \approx 2\gamma = 2\ J/m^2$$

For ductile materials such as steel, the plastic dissipation term dominates

$$G \approx G_p = 1000\ J/m^2$$

For polymers close to the glass transition temperature, we have intermediate values of G between 2 and 1000 J/m^2 [6].

D. Stress intensity Factor

The stress intensity factor K, is used in mechanics to predict the stress state ("stress intensity") near the tip of a crack or notch caused by a remote load or residual stresses. It is a theoretical construct usually applied to a homogeneous, linear elastic material and is useful for providing a failure criterion for brittle materials, and is a critical technique in the discipline of tolerance. The concept can also be applied to materials that exhibit small-scale yielding at a crack tip. The magnitude of Stress intensity factor depends on sample geometry, the size and location of the crack or notch, and the magnitude and the modal distribution of loads on the material [7].

E. Fracture Toughness

In materials science, fracture toughness is a property which describes the ability of a material to resist fracture, and is one of the most important properties of any material for many design applications [8].

F. Prediction of Failure in Pressure Vessels

In practice, pressure vessels have a multi-axial stress situation, where failure is not governed by the individual components of stress but by some combination of all the stress components. Many theories of failure have therefore been developed to predict the onset of failure in these complex systems. Among the failure theories, Von Mises criterion is generally accepted to be better suited for common pressure vessels as it is found to be more accurate. Terica’s criterion is commonly used for the design by analysis procedure for two reasons. It is more conservative and it is considered easier to apply. However, with the availability of computers, it has also made it easier to apply the Von Mises criterion. All the same, failure theories approach does not consider the effect of cracks or flaws, which can significantly degrade structural integrity and therefore cannot be applied to deal with failure prediction in cases where fracture of a component is likely to occur and therefore, fracture mechanics method can be adopted instead [3].

G. Leak before break Test

Leak before burst describes a pressure vessel designed such that a crack in the vessel will grow through the wall, allowing the contained fluid to escape and reducing the pressure, prior to growing so large as to cause fracture at the operating pressure Many pressure vessel standards, including the ASME Boiler and Pressure Vessel Code and the AIAA metallic pressure vessel standard, either require pressure vessel designs to be leak before burst, or require pressure vessels to meet more stringent requirements for fatigue and fracture if they are not shown to be leak before burst pressure vessel is to be designed using the leak-before-break criterion based on the circumferential wall stress and plane-strain fracture toughness. The design stress is restricted by the yield strength and a safety factor (SF). Derive expressions for a) the critical crack size and b) the maximum allowable pressure when the crack size is equals to the vessel thickness. It is often advantageous to design pressure containing plant, such as pipework, tubes, vessels, and boilers, on the basis of leak-before-break. This means that partial failures which occur by sub-critical mechanisms (fatigue crack growth, stress corrosion cracking etc.) are detected by loss of pressure in the plant before final catastrophic fracture occurs. This requires a crack to grow in a stable manner through the wall of the component and cause a detectable leak and consequent loss of pressure. This indication of a partial failure allows the plant to be shut down in a controlled manner and repairs/replacement carried out. If it can be demonstrated that a leak-before-break situation exists, other useful benefits may occur by Supplementing the primary structural integrity safety case. Alleviating some of the responsibility of non-destructive testing for ensuring safety. Permitting a reduction in number of restraints engineered into a pipework system to control pipe whip on failure.

3. Problem statement

The proposed work aims at fracture analysis of pressure vessel by using theoretical and finite element method using software.

The problem statement is follows:

A cylindrical pressure vessel with closed ends has a radius R = 1 m and thickness t =40 mm and is subjected to internal pressure p_i. The vessel must be designed safely against failure by yielding (according to the von Mises yield criterion) and fracture.

Three steels with the following values of yield stress σ_y and fracture toughness K_{ic} are available for constructing the vessel.

Fracture of the vessel is caused by a long axial surface crack of depth a. The vessel

should be designed with a factor of safety S = 2 against yielding and fracture. For each steel we have:

a) Calculate the maximum permissible crack depth ac for an internal pressure

$$P_i = 12 \text{ MPa};$$

b) Calculate the failure pressure Pc for a minimum detectable crack depth

$$a_c = 1 \text{ mm}.$$

c) Select the best alternative material from given option.

The proposed work aims at fracture analysis of pressure vessel by using theoretical and finite element method using software.

Table 1

Steel	σ_y (MPa)	K_{IC} $(MPa\sqrt{m})$
4340	860	100
4335	1300	70
350 Maraging	1550	55

4. Methodology

A. *Leak before break methodology*

One of the earliest methods to address LBB in pressure vessels was due to Irwin where the LBB was postulated to occur due to an axial flaw in a pressure vessel, if the flaw length was less than twice the shell thickness. This implies that the crack driving force in the radial direction would exceed that in the axial location under these conditions.

Subsequently this criterion was modified by other researchers by including free surface effects, bulging effects and toughness differences in through-wall crack versus surface growth directions. As noted by Witkowski LBB procedures and analyses vary from one industry to another depending on the level of risk and the nature of loading experienced during operation.

In the pressure vessel, the axial flaws tend to be problematic because of the existence of large compressive longitudinal stresses. In the nuclear industry the concepts of LBB has been applied to pressure vessel for the purpose of eliminating equipment used for restraining pressure vessel whipping from a postulated pressure vessel rupture event. The concern in this application is with the above where circumferential flaws are historically more prevalent than the axial flaws. In the LBB: approach it is desirable to detect small amounts of leakage at normal operating conditions so that the leakage size flaw will be stable at transient stresses.

It is also essential that there not be any sub-critical crack growth mechanism that could cause long surface flaws to occur. Such long surface flaws could lead to failure under the transient loads without any leakage warning.

Application of the LBB concept thus requires reliable leak detection systems and verified leak rate estimation techniques. An important issue is to determine the condition under which piping would leak sufficiently prior to break so that an operator action could be taken before a catastrophic failure occurs [12].

1) The criteria for selection of materials is as follows,

$$P_i \le \left\{\frac{{K_{IC}}^2}{\sigma_y}\right\}$$

Where,

P_i- Internal Pressure
R- Radius of Cylindrical Pressure Vessel
K_{IC}- Critical Stress Intensity Factor.
σ_y- Yield Strength [11]

2) Crack stability:

$$\sigma_h \le \left\{\frac{K_{IC}}{\sqrt{\frac{\pi t}{2}}}\right\}$$

Where; σ_h= Hoop Stress [10]

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5. Numerical analysis of unfired pressure vessel

A. *Solution of problem*

Design of the pressure vessel necessitates analysis of the stress field coupled with a failure criterion. As such, the von Mises yield criterion will be used for design against failure by yielding and the critical stress intensity factor criterion will be used for design against unstable crack growth.

1) *Stress field*

A material element of the vessel is subjected to a hoop stress σ_h and a longitudinal. stress given by

$$\text{Hoop stress} = \sigma_h = \frac{p_i R}{t} = \frac{12*1}{0.04} = 300 \text{ MPa} \quad (1)$$

$$\text{Longitudinal stress} = \sigma_z = \frac{p_i R}{2t} = \frac{12*1}{0.04*2} = 150 \text{ MPa} \quad (2)$$

2) *Failure by yielding*

The von Mises yield criterion for a two-dimensional stress field with principal stresses σ_h and σ_z takes the form

$${\sigma_h}^2 - \sigma_h\sigma_z + {\sigma_z}^2 = \left\{\frac{\sigma_y}{S}\right\}^2 \quad (3)$$

From eqn. (1), (2) & (3) we obtain,

$$P_{max} = \frac{\sigma_y * t}{\sqrt{3}\, R} \quad (4)$$

$P_c = 23.2*10^{-3} * \sigma_y$

Eqn. (4) gives the maximum pressure the vessel can withstand without failure by yielding,

Table 2
Maximum pressure without failure

Steel	Maximum pressure without failure
4340	19.9
4335	30.0
350 Marging	35.8

Table 3
Result for failure Pressure with min & max. crack

Steel	Failure pressure (MPa) when crack length is minimum i.e. a=1mm	Failure pressure (MPa) when crack length is maximum i.e. a=40mm
4340	10.15	5.038
4335	7.05	3.53
350 Marging	5.543	2.77

Table 4
Applying criteria for material selection

Steel	Internal pressure	$P_i \leq \left\{\frac{{K_{IC}}^2}{\sigma_y}\right\}$	Hoop stress in Mpa	$\leq \left\{\frac{K_{IC}}{\sqrt{\frac{\pi t}{2}}}\right\}$ in (MPa)
4340	12	11.627≈ 12	300	400
4335	12	3.7693	300	280
350 Marging	12	0.1947	300	220

Table 5
Results

Steel	P_{max} without failure	P_f when a=1mm	P_f when a=40mm	% loss of pressure	$P_i \leq \left\{\frac{{K_{IC}}^2}{\sigma_y}\right\}$	$\sigma_h \leq \frac{K_{IC}}{\sqrt{\frac{\pi t}{2}}}$
4340	19.9	10.15	5.038	48.99	Satisfies	Stable
4335	30.0	7.05	3.53	76.5	Not Satisfies	unstable
350 Marging	35.8	5.543	2.77	84	Not Satisfies	unstable

3) *Failure by unstable crack growth*

Consider a long axial surface crack of depth a in the vessel. The stress intensity factor at the crack tip is

$$K_I = 1.12\ \sigma_h \sqrt{\pi a} \tag{5}$$

The fracture condition is

$$K_I = \frac{K_{IC}}{2} \tag{6}$$

From eqn. (5) & (6) we obtain,

$$\mathrm{P_C} = \frac{\mathrm{tK_{IC}}}{2.24\sqrt{\mathrm{\pi aR}}} \tag{7}$$

Table of results for failure pressure is given below

4) *Applying Criteria for Selection of material*

$$P_i \leq \left\{\frac{{K_{IC}}^2}{\sigma_y}\right\}$$

$$\sigma_h \leq \left\{\frac{K_{IC}}{\sqrt{\frac{\pi t}{2}}}\right\}$$

Table of applying criteria for material selection.

6. Conclusion

From the numerical analysis, we conclude that,

- The steel 4340 satisfies the condition of maximum pressure safely carried by material.
- It also satisfies the condition of stable crack growth i.e. means the catastrophic failure of pressure vessel will not occurs.
- In case of material like steel 4335 and 350 marging does not fulfill the criteria of material selection for pressure vessel and which results into an unstable crack growth which will lead to catastrophic failure of material.
- The steel 4340 will be the proper, suitable, reliable, safe material for unfired pressure vessel

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