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Standard Domination Number of Jump Graphs

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***Abstract*: The graph theory properties of $\gamma'[J(G)]$ and its exact values for some standard graphs. The relation between $\gamma'[J(G)]$ with other parameter are also discussed. We also present some results domination graph $M_{ed}(J(G))$ for some graphs.**

***Keywords*: domination number, jump graph.**

1. Introduction

Over the past decay, the domination number of jump graph is one of the new concepts in graph theory which has attracted several researchers because of various applications such as linear algebra and optimization, communication networks, social sciences and in the existing literature. The concept of edge domination was introduced by Mitchell and Hedetniemi [1] and it was explored by many researchers. Recently Arumugam and Velammal have discussed the edge domination in graphs [2]. Herein, we investigate domination number in jump graph.

2. Mathematic formulation

Definition: 1.1

The line graph $L(G)$ of G have the edges of G and vertices be adjacent in $L(G)$. Let the complement of line graph $L(G)$ as the jump graph $J(G)$ of G. The jump graph $J(G)$ is defined on $E(G)$ whose vertices are adjacent.

Definition: 1.2

The set $A \subseteq E[J(G)]$ be the dominating set of $J(G)$, if every edge not in A is adjacent to a edge in D. The dominal number of jump graph, denoted by $\gamma'[J(G)]$, is the minimum cardinality of a dominating set in $J(G)$.

Theorem: 1.1

1. At path P_p, with $p \geq 5$, $\gamma'[J(P_p)] = 2$.
2. At Cycle C_p, with $p \geq 5$, $\gamma'[J(C_p)] = 2$
3. At Complete graph, K_p with ≥ 5 , $\gamma'[J(K_p)] = 3$.
4. At complete bipartite graph K_{mn},
$$\gamma'[J(K_{ij})] = \begin{cases} 2 \text{ for } K_{2,n} & \text{where } i > 2 \\ 3 \text{ for } K_{m,n} & \text{where } j, i \geq 3 \end{cases}$$
5. At wheel W_p, $\gamma'[J(W_p)] = \begin{cases} 3 \text{ for } p = 5,6 \\ 2 \text{ for } p \geq 7 \end{cases}$

Theorem: 1.2

For any connected graph G with diameter, $diam(G) \geq 2$, $\gamma'[J(G)] \geq 2$

Proof:

Let a, b be a path of maximum distance in G.
Then, $d(a, b) = diam(G)$.
We can prove the theorem with the following cases.

Case (1):

For $diam(G) = 2$, Choose a vertex v_1 of eccentricity 2 with maximum degree among others.
Let $E_1 = \{e_1^1, e_2^1, \dots\}$ corresponding to the elements of $\{v_1, v_2, \dots\}$ for $J(G)$. Every edge $v \notin E_1$ is adjacent to a edge in E_1. Hence E_1 is a minimum dominating set. The jump graph is $\gamma'[J(G)] > 2$.

Case (2):

For $diam(G) > 2$, let v_1 be the vertex which is adjacent to v and v_2 with respect to u.
Let $\{v_1, v_2\} \subseteq V(G)$ corresponds to $\{e_1^1, e_2^1\} \subseteq E(J(G))$.
Since these edges $\{e_1^1, e_2^1\}$ are adjacent to all other edges of $E(J(G))$, it follows that $\{v_1, v_2\}$ becomes a minimum dominating set. Hence, $\gamma'[J(G)] = 2$.
From this, we show G $\gamma'[J(G)] \geq 2$.

Theorem: 1.3

Let T be the tree with diameter is maximum, $\gamma'[J(T_d)] = n$.

Proof:

When diameter of the jump graph is minimum, then it is dislocated.
Let uv be a path of maximum length in a tree T where diameter is greater than 3. Let e_i be the pendent vertex adjacent to u and e_k be the pendent edge adjacent to v. The edge set $e_i, i = 1,2,3, \dots n$, of $J(T_d)$ corresponding to the vertices in T will form the dominating set in $J(T_d)$. When $E[J(T)]$ are adjacent with $e_i, i = 1,2,3, \dots n$ it form a minimum dominating set.
Hence $\gamma'[J(T_d)] = n$.

Theorem: 1.4

For any connected (p, q) graph G, $\gamma'[J(G)] \leq p - \Delta(G)$
Where,
$\Delta(G)$ is the maximum degree of G.

Proof:

Let the set of edges be $E = \{e_1, e_2, \dots, e_i\}$, and
$$E_1 = E - e$$
Where,
e_1 be the edge shows maximum.
When, $E(G) = V[J(G)]$.
Consider $I = \{v_1, v_2, \dots, v_i\}$ as the set of vertices adjacent to e_1.
Let $H \subseteq E[J(G)]$ be the set of edges of $J(G)$ such that $H \subseteq V - I$. Then H itself forms a minimally dominating set. Therefore $\gamma'[J(G)] \leq |V| - |I|$.
Hence $\gamma'[J(G)] \leq p - \Delta(G)$.

Theorem: 1.5

For any connected (p, q)-graph G, $\gamma_{sl}((T_d)) \leq \left\lceil \frac{p}{3} \right\rceil$.

Proof:
Let $D = \{v_1, v_2, \ldots, v_n\} \subseteq V(L(G))$ be the minimum split line dominating set.
In case, $|V(L(G)) - D| = 0$. Then result shows immediately. When, $|V(L(G)) - D| \geq 3$, then $V(L(G)) - D$ contains at least two vertices such that $2n < p$. Clearly, it follows that $\gamma_{sl}((T_d)) \leq \lceil p/3 \rceil$

Theorem: 1.6
When (p, q) be the tree (T_d), $\gamma_{sl}((T_d)) \leq q - \Delta'((T_d))$.

Proof:
Let $A = \{v_1, v_2, \ldots, v_n\} \subseteq V(L((T_d)))$ be the adjoint vertices.
The vertices $A_1 = \{u_1, u_2, \ldots, u_m\} \subseteq V(L((T_d))) - A$ (i.e) $dist(u_i, v_j) \geq 2,\ \forall\, u_i \in A_1, v_j \in A, 1 \leq i \leq m, 1 \leq j \leq n$.
Then, evidently $S = A \cup A_1$ shows split line dominating set with respect to (T_d). Otherwise, if $A \not\subset V(L((T_d)))$, then vertices

$S = A_1$ (i.e) $N[S] = V(L((T_d)))$ and
$\langle V(L((T_d))) - S \rangle$ is dislocated.

Evidently, S shows a minimal split line dominating set of (T_d). Since for any tree (T_d), there exists at least one edge $e \in E((T_d))$ with $\deg(e) = \Delta'((T_d))$,
we obtain $|S| \leq |E((T_d))| - \Delta'((T_d))$.
Therefore, $\gamma'_{sl}((T_d)) \leq q_j - \Delta'((T_d))$.

3. Conclusion

Here, we present some results on these types of graphs and also we discuss about the domination number of jump graph for some standard notation.

References

[1] S. Mitchell and S. T. Hedetniemi, “Edge domination in trees,” *Congressus Numerantium*, vol. 19, pp. 489-509, 1977.