

Cordially Divisor Rhombus Grid Graph

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Abstract: In this paper, we introduce some album grid divisor cordial graphs. Further, we investigate the graphs obtained by the identification of some vertices of a divisor cordial graphs to certain rhombus grid graphs are divisor cordial.

Keywords: Divisor Cordial graph, Vertices, edges, album.

1. Introduction

Number Theory is a fascinating subject in mathematics. It has so many interesting concepts. The concepts of primality and divisibility play an important role in Number Theory.

Definition 1.

Let $G = (V, E)$ be the function of $f:V$ is denoted by the set $\{0,1\}$ with an each edge xy , is ascribed by the label 1 if $f(x)$ divides $f(y)$ or $f(y)$ divides $f(x)$ and 0 or other else, then the edges are labeled with 0 and 1 differ by at most 1 [1].

For each edge xy , assign the label 1 if either $[f(x)]^2 | f(y)$ or $[f(y)]^2 | f(x)$ and the label 0 otherwise. f is called a rectangular divisor cordial labeling if $|e_c(0) - e_c(1)| \leq 1$. A graph with a rhombus divisor cordial labeling is called a rhombus divisor cordial graph.

Definition: 1.1

One edge union of cycles having same length is called a notebook. By common, the edge is said to be the base of the notebook. If we assume t copies of cycles of length m then the notebook is denoted by $R_m^{(t)}$. Note that $R_m^{(t)}$ has $(m-2)t+2$ vertices and $(m-1)t+1$ edges.

Theorem: 1.2

Let P be the rhombus grid pages is divisor cordial.

Proof:

Let R be the rhombus grid. Note that it has $2t+2$ vertices and $3t+1$ edges. Label the vertices of common edge by 1 and 2. Then label the vertices of the edges which are parallel to common edge as given below.

Example: 1.3

Let us consider the rhombus grid. Note that it has 6 vertices and 7 edges.

Here, we have $e_f(0) = 3$ and

$$e_f(1) = 4.$$

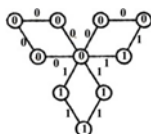


Fig. 1. Rhombus grid

In the first page, label the numbers 4 and 3, second page 5 and 6, Since 1 divides all the integers it contributes $t+1$ to $e_f(1)$, 2 divides all the even integers it contributes $\frac{t}{2}$ to each $e_f(0)$ and $e_f(1)$.

When t is even,

$$e_f(0) = \frac{t+1}{2} \text{ and}$$

$$e_f(1) = \frac{t-1}{2}.$$

When t is odd, $m \neq m+1$ for any integer $m > 1$, the parallel edges are assigned t to $e_f(0)$.

Consequently,

Case (1) if t is even,

$$e_f(0) = \frac{3t}{2} \text{ and}$$

$$e_f(1) = \frac{3t}{2} + 1 \text{ and}$$

Case (2) if t is odd, then

$$e_f(0) = \frac{3t+1}{2} \text{ and}$$

$$e_f(1) = \frac{3t+1}{2}$$

Thus, $|e_f(0) - e_f(1)| \leq 1$. As a consequence, R shows divisor cordial.

Corollary: 1.4

A rhombus grid with even number is divisor dominated cordial but not strict.

Proof:

The rhombus grid N is divisor dominated cordial graph. If we interchange the labels of second page, then N becomes non divisor dominated cordial.

Theorem: 1.5.

Let G be a divisor cordial graph and R be the rhombus grid. Then $G_R * R$ is divisor cordial.

Proof:

Let us assume G is a divisor cordial graph of order p and size q and the vertices labeled 1 and 2 are not adjacent. Here, f^* be the divisor.

Let R be a rhombus grid labeled at f_R . Now identify the vertices labeled 1 and 2 in G to the vertices of common edge of R . We already proved that $G_N * R$ be the divisor cordial. Let f be the labeling of $G_N * R$.

Case (i): if p is even.

Since G is divisor cordial, we have $e_{f^*}(0) = e_{f^*}(1) = \frac{t}{2}$.

Case (ii): if t is even, the vertices of the parallel edges in R to edge of the vertices are labeled 1 and 2 as follows.

If n is even,

$$e_f(0) = \frac{m}{2} + \frac{3t}{2} \text{ and}$$

$$e_f(1) = \frac{m}{2} + \frac{3t}{2} + 1.$$

If t is odd, divisor dominated cordial which implies

$$e_{f^*}(0) = \frac{m+1}{2} \text{ and}$$

$$e_{f^*}(1) = \frac{m+1}{2}$$

In all cases, $|e_f(0) - e_f(1)| \leq 1$. So, $G_N * R$ is divisor cordially.

Example: 1.6

Consider the following divisor cordial graph G

It has even order and even size. Note that the vertices labeled 1 and 2 are not adjacent.

Now, we shall connected with the album P with rectangular pages to G

Here, we see that $e_f(0) = 4$ and $e_f(1) = 4$

This example illustrates the subcase(a) of Case (i) for even order of G .

Next, we shall explain the subcase (a) of Case (ii) by the following example.

Example .1.7.

Consider the following divisor cordial graph G of odd size.

Here, $m = 5$ and $n = 4$ and $e_f(0) = 2$ and $e_f(1) = 3$

Then rhombus grid is attached with pages as given below.

Here, we see that $e_f(0) = 4$ and $e_f(1) = 5$

Theorem .1.8.

Let G be a divisor cordial graph and R be a rhombus grid and let e be the common edge of R . Then $G * G (R - e)$ is divisor cordial.

Proof:

Here the vertices labeled 1 and 2 in G are adjacent.

Case (i): (a) if m is even, t is even.

$$\text{Here } e_f(0) = e_f(1) = \frac{m}{2} + \frac{3t}{2}.$$

(b) if m is even, t is odd

$$\text{Here } e_f(0) = \frac{m}{2} + \frac{3t}{2} \text{ and } e_f(1) = \frac{m}{2} + \frac{3t}{2}.$$

Case (ii): (a) if m is odd, t is even.

$$\text{Here } |e_f(0) - e_f(1)| \leq 1.$$

(b): m is odd, t is odd.

From this, we were interchanging the labels of the vertices of R .

Then, we have $|e_f(0) - e_f(1)| \leq 1$.

Thus, in all the cases we see that $G * G (R - e)$ is divisor cordial.

2. Conclusion

The rhombus grid divisor cordial graphs are discussed and also, we can prove that the graphs obtained by the identification of some vertices of a divisor cordial graphs to certain rhombus grid graphs are divisor cordial.

References

- [1] D. M. Burton, "Elementary Number Theory," Second Edition, Wm. C. Brown Company Publishers, 1980.