International Journal of Research in Engineering, Science and Management
Volume-2, Issue-9, September-2019
www.ijresm.com | ISSN (Online): 2581-5792

Sum Divisor Cordial Graph

R. Elambarithi[1], R. Anitha[2]
[1]M.Phil. Scholar, Dept. of Mathematics, Ponnaiyah Ramajayam Inst. of Science and Tech., Thanjavur, India
[2]Assistant Professor, Dept. of Mathematics, Ponnaiyah Ramajayam Inst. of Science and Tech., Thanjavur, India

***Abstract*: In this paper, we investigate about a sum divisor cordial graph, we prove that path, comb, jewel, crown, are sum divisor cordial graphs.**

***Keywords*: Divisor, cordial graph, path.**

1. Introduction

The divisor cordial labeling is a variant of cordial labeling. It is very interesting to investigate graph or graph families which are divisor cordial as all the graphs do not admit divisor cordial labeling [1]. Here, we proved that path, jewel graph and crown graph are divisor cordial graphs.

2. Mathematic Formulation

Definition: 1.1
Let $G = (V(G), E(G))$ be a simple graph and $f: V(G) \rightarrow \{1,2,\cdots,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $2|(f(u) + f(v))$ and the label 0 otherwise.The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph that admits a total divisor cordial labeling is named a total divisor cordial graph.

Definition: 1.2
The comb $P_n \odot K_n$ is the graph obtained from a path by attaching a pendant edge to each vertex of the path.

Example: 1.3
The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is
$E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv: u \in V(G_1), v \in V(G_2)\}$

Example: 1.4
The jewel J_n is the graph with vertex set
$V(J_n) = \{u, v, x, y, u_i: 1 \leq i \leq n\}$ and
Edge set
$E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

Example: 1.5
For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Theorem 1.6
The path P_n is sum divisor cordial graph.

Proof:
Let P_n be a path with consecutive vertices $v_1, v_2, \cdots, v_n$.Then P_n is of order n and size $n - 1$.
Define $f: V(P_n) \rightarrow \{1,2,\ldots,n\}$ as follows:

Case (i): n is odd

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 0,1 (mod\ 4) \\ i+1 & \text{if } i \equiv 2 (mod\ 4) \\ i-1 & \text{if } i \equiv 3 (mod\ 4) \end{cases} \text{ for } 1 \leq i \leq n$$

Case (ii): n is even

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 1,2 (mod\ 4) \\ i+1 & \text{if } i \equiv 3 (mod\ 4) \\ i-1 & \text{if } i \equiv 0 (mod\ 4) \end{cases} \text{ for } 1 \leq i \leq n$$

In both cases the induced edge labels are

$$f^*(v_i v_{i+1}) = \begin{cases} 1 & if\ 2\,|(f(v_i) + f(v_{i+1})) \\ 0 & otherwise \end{cases}$$

We observe that,

$$e_f(0) = \begin{cases} \frac{n-1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$$

$$e_f(1) = \begin{cases} \frac{n-1}{2} & if\ n\ is\ odd \\ \frac{n-2}{2} & if\ n\ is\ even \end{cases}$$

Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, the path P_n is sum divisor cordial graph.

Theorem 1.8
The comb is sum divisor cordial graph.
Proof:
Let G be a comb obtained from the path $v_1, v_2, \cdots, v_n$ by joining a vertex u_i to v_i for each i =1, 2,⋯,n. Then G is of order $2n$ and size $2n - 1$.
Define $f: V(G) \rightarrow \{1,2,\cdots,2n\}$ as follows:

$$f(v_i) = 2i - 1; \ 1 \leq i \leq n$$
$$f(u_i) = 2i; \ 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(v_i v_{i+1}) = 1; \ 1 \leq i \leq n-1$$
$$f^*(v_i u_i) = 0; \ 1 \leq i \leq n$$

We observe that, $e_f(0) = n$ and $e_f(1) = n - 1$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, the comb is sum divisor cordial graph.

Theorem 1.9

The jewel J_n. is sum divisor cordial graph.

Proof:

Let $G = J_n$. Let $V(G) = \{u, v, x, y, u_i : 1 \le i \le n\}$ and $E(G) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \le i \le n\}$. Then G is of order $n + 4$ and size $2n + 5$.

Define $f : V(G) \to \{1,2,\cdots, n+4\}$ as follows:

$f(u) = 1;$
$f(v) = 2;$
$f(x) = 3;$
$f(y) = 4;$
$f(u_i) = i + 4;\ 1 \le i \le n.$

Then, the induced edge labels are

$f^*(ux) = 1;$
$f^*(uy) = 0;$
$f^*(xy) = 0;$
$f^*(vx) = 0;$
$f^*(vy) = 1;$
$f^*(uu_{2i-1}) = 1;\ 1 \le i \le \left\lceil\frac{n}{2}\right\rceil$
$f^*(uu_{2i}) = 0;\ 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$
$f^*(vu_{2i-1}) = 0;\ 1 \le i \le \left\lceil\frac{n}{2}\right\rceil$
$f^*(vu_{2i}) = 1;\ 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$

We observe that, $e_f(0) = n + 3$ and $e_f(1) = n + 2$.

Thus, $|e_f(0) - e_f(1)| \le 1.$

Hence, J_n is sum divisor cordial graph.

Theorem 1.10

The gear G_n is sum divisor cordial graph.

Proof:

Let $G = G_n$. Let $V(G) = \{v, u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{vv_i, v_iu_i : 1 \le i \le n; u_iv_{i+1} : 1 \le i \le n - 1; u_nv_1\}$. Then G is of Order $2n + 1$ and size $3n$.

Define $f : V(G) \to \{1,2,\cdots, 2n + 1\}$ as follows:

$f(v) = 1;$
$f(v_{2i-1}) = 4i - 1;\ 1 \le i \le \left\lceil\frac{n}{2}\right\rceil$
$f(v_{2i}) = 4i;\ 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$
$f(u_{2i-1}) = 4i - 2;\ 1 \le i \le \left\lceil\frac{n}{2}\right\rceil$
$f(u_{2i}) = 4i + 1;\ 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$

Then, the induced edge labels are

$f^*(vv_{2i-1}) = 1;\ 1 \le i \le \left\lceil\frac{n}{2}\right\rceil$
$f^*(vv_{2i}) = 0;\ 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$
$f^*(v_iu_i) = 0;\ 1 \le i \le n$
$f^*(u_iv_{i+1}) = 1;\ 1 \le i \le n - 1$
$f^*(u_nv_1) = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$

We observe that, $e_f(0) = \left\lceil\frac{3n}{2}\right\rceil$ and

$e_f(1) = \left\lfloor\frac{3n}{2}\right\rfloor$

Thus, $|e_f(0) - e_f(1)| \le 1.$

Hence, G_n is sum divisor cordial graph

3. Conclusion

The divisor cordial labeling is a variant of cordial labeling. Here, we discuss about the sum divisor cordial graph, and also prove that path, comb, jewel, crown, are sum divisor cordial graphs.

References

[1] S. K. Vaidya and N. H. Shah, "Some Star and Bistar connected Divisor Cordial Graphs," *Annals of Pure and applied math.*, vol. 3, no. 1, pp. 67-77, 2013.