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Working Vacations on Multi-Server Retrial Queue

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***Abstract*: In this paper, we investigate the multi-server retrial queue with vacations under various vacation policies can be modeled by Markov process with matrix components. The stationary distribution of the queueing systems with retrials and work vacations on the steady state space.**

***Keywords*: Multi-Server, Retrial queue**

1. Introduction

Queueing theory is one such area where probability models can effectively be used. Queueing theory was developed to provide models to predict the behavior of systems that attempt to provide service for randomly arising demands; not unnaturally, then, the earliest problems studied where those of telephone traffic congestion [1]. Here, we analyze the multi-server retrial queue with working vacation.

2. Mathematical formulation

The steady state probabilities of the multi-server retrial queue with working vacations.

Let A*, B* and C* be matrices with the elements A* (x, y), B* (x, z) and C* (x, z), respectively.
Then the matrices can be written as

$$A^* = A = \begin{bmatrix} 0 & \lambda_B & \theta & 0 \\ \mu_r & 0 & 0 & \theta \\ 0 & 0 & 0 & \lambda_B \\ 0 & 0 & \mu_b & 0 \end{bmatrix}$$

$$B^* = B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_B \end{bmatrix}$$

$$C^* = C = \begin{bmatrix} 0 & \rho & 0 & 0 \\ 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & \rho \\ 0 & 0 & 0 & \lambda_B \end{bmatrix}.$$

At $N(t) = 0$, when there is no customer in the orbit, the arrival of a new customer may following reasons

When the server is not occupied, then the server changes to the busy state (i.e.: $S(t)$ changes either from 0 to 1), while $N(t)$ remains unchanged.

When the server is occupied, ($S(t)$ = 1 or 3), then the customer goes into the orbit.

After departure of a customer, the server becomes free ($S(t)$ changes either from 1 to 0 or from 3 to 0) and $N(t)$ remains unchanged.

The status change of the server (i.e.: the end of the vacation), then $S(t)$ changes from 1 to 3.

The generator matrices as shown below:

$$A^*_0 = \begin{bmatrix} 0 & \lambda_B & \theta & 0 \\ \mu_v & 0 & 0 & \theta \\ 0 & 0 & 0 & \lambda_B \\ 0 & 0 & \mu_b & 0 \end{bmatrix}$$

$$B^*_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_B \end{bmatrix}$$

The multi-server retrial queue at any time t is denoted by

$S^*(t)$ = 0 the server is free at time t
1 the server is busy at time t

$N^*(t)$ be the number of customers in the orbit at time t.

The M/M/c retrial queue is a continuous time discrete state Markov process,

$\{S^*(t), N^*(t)\}$, on the state space $\{(x, y) : x = 0,1, y \geq 0\}$.

The transition rates of the matrices can be written as

$$A_y^* = A^* = \begin{bmatrix} 0 & \lambda_B \\ \mu_1 & 0 \end{bmatrix}$$
$$B_y^* = B^* = \begin{bmatrix} 0 & 0 \\ 0 & \lambda_B \end{bmatrix}$$
$$C_q^* = C^* = \begin{bmatrix} 0 & \rho \\ 0 & 0 \end{bmatrix} \gamma_q \geq 1$$

The polynomial of the retrial M/M/c queue matrix is obtained as

$$Q^*(x) = \begin{bmatrix} Q_{22}x & (\lambda_B x)\rho x^2 \\ \mu_b x & Q_{33}x \end{bmatrix}$$

$$\varphi_j = \sum_{1=3}^{4} a_i^* x_i^* \Psi_i^*$$

The eigen values namely x_3, x_4 and x_5, which form the subset of the eigen values are associated with the retrial M/M/c queue with working vacations.

The eigen vector x_3 of

$Q^*(x)$ is $\Psi_3^* = [1, 0]$,

while

$\Psi_4^* = [\mu_V, \rho + \lambda_B]$ is the corresponds to eigenvectors of eigen value x_4.

where

a_i^* is the coefficients, which can be determined from the balance equation for $N^*(t) = 0$ and the normalization equation

$$a_3^* = \frac{\alpha\mu_b - \alpha\lambda - \lambda^2}{(\alpha+\lambda)\mu_b}$$
$$a_4^* = \frac{\alpha\lambda\mu_b - \alpha\lambda - \lambda^2}{(\alpha+\lambda)\mu_b^2}$$

The probability of the number of customers in the orbit, given the server is busy, is

$$N(z) = \sum_{j=0}^{\infty} x_4^j z^j \, a_4^*(\alpha+\lambda) = \frac{(\alpha+\lambda)a_4^*}{1 - zx_4}$$

2.1 Steady state analysis

The steady state space is

χ = m:

Where, m = {0 , 1 , 2 , . . .}

The generator form is

$$Q^* = \begin{pmatrix} B_0 & A_0 & 0 & 0 & 0 & \dots \\ C_1 & B_1 & A_1 & 0 & 0 & \dots \\ 0 & C_2 & B_2 & A_2 & 0 & \dots \\ 0 & 0 & C_3 & B_3 & A_3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

The matrices A_n ; B_n and C_n are given by

$$A_m = \begin{pmatrix} 0 & \\ & A_j \end{pmatrix}$$

$$C_m = \begin{pmatrix} 0 & C_0 & & \\ & 0 & \ddots & \\ & & \ddots & C_{j-1} \\ & & & 0 \end{pmatrix}, m \geq 1$$

$$B_m = \begin{pmatrix} B_{00} & A_{01} & 0 & 0 & 0 \\ C_{10} & B_{11} & A_{12} & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & B_{j-1,j-2} & B_{j-1,j-1} & B_{j-1,j} \\ \vdots & \vdots & \vdots & B_{j,j-1} & B_{j,j} \end{pmatrix}, m \geq 0$$

Let

$N^*(t)$ be the number of customers in orbit and

$S^*(t)$ be the state of the server at time t.

$I_a(t)$ be the phase of the arrival process and $J_s(t)$ be the server state at time t defined by

$I_s(t) = 0$, the servers are available

i, the phase of working vacation time is of i, $1 \leq i \leq w$.

The Markov process

$Z^* = \{Z^*(t), t \geq 0\}$ with $Z^*(t)$

$= (N^*(t), S^*(t), I_a(t), I_s(t)$ is a continuous time discrete state Markov chain on the state space

$$S^* = \sum_{m=0}^{\infty} m,$$

Where,

m = $\{(m, j, i, k) : 0 \leq j \leq J, 1 \leq i \leq l, 0 \leq 6 \leq w\}$; $m \geq 0$.

The matrix components of A_m and C_m of the generator Q of Z^* are given as

$A_m = D \otimes I_{w+1}$

$Bj_{,j+1} = D \otimes I_{w+1}$

$Cj = \gamma_n I_{w+1}$

Where,

$$M_0(\mu)\, p = \begin{pmatrix} 0 & \mu\delta \\ 0 & \mu I_w \end{pmatrix}$$

$$M_1(\mu) = \begin{pmatrix} \mu & 0 \\ 0 & \mu I_w \end{pmatrix}$$

2.2 Computation of the matrix R

The matrix R as a limit of R_n, n = 0 ; 1 ; 2 ; . . ., Where, $R^* = 0$, $R_n = (A_0 + R^* A_2)(-A_1)^{-1}$, N = 1,2
It is also recommended for computing R.

2.3 Numerical results

First, we depict the behavior of y_0 (K, N)1 = Pr(X_0 = 0) for the convergence of stationary distribution as truncation levels K and N increase.

The parameters are
μ = 1:0, s = 10, λ = 10:0, θ = 0:5, γ_n = 10n, (n = 0; 1 ; 2; . . .) and the probabilities $a_i(j)$, $b_i(j)$, $c_i(j)$ are as follows:

$\alpha_0^* = 0.02$, $\alpha_0 = 0.05$, $\beta_0^* = 0.03$, $\beta_0 = 0.05$, $\gamma_0^* = 0.2$, $\gamma_0 = 0.25$,
$\alpha_1^* = 0.28$, $\alpha_1 = 0.35$, $\beta_1^* = 0.37$, $\beta_1 = 0.5$, $\gamma_1^* = 0.8$, $\gamma_1 = 0.75$,
$\alpha_2^* = 0.7$, $\alpha_2 = 0.6$, $\beta_2^* = 0.6$, $\beta_2 = 0.45$.

The values K, y_0 (K,N)1 approaches to a constant as N increases and for large N, y_0 (K,N)1 decreases monotonically as K increases. Thus we can see that y_0 (K,N)1 converges to a constant.

Now we illustrate the algorithmic for stationary distribution were investigate to performance the characteristics. Let N_0 and N_1 be the number of customers in orbit and service requests, respectively in stationary state and $S_0 = E[X_0]$, $L_1 = E[X_1]$. The probability P_r and the loss probability P_L are given by the formulae,

$P_r = P(X_1 \geq s) = x_{ij}$,
$P_l = \frac{1}{\lambda b}\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \lambda a_0(j) + \gamma_i b_0(j) + \theta_j c_0(j) x_{ij}\right)$

3. Conclusion

We introduce the new multi-server retrial queuing system with working vacations for customer's orbit. They are exponentially distributed with intensity depending on the number of customers in the orbit. Efficient algorithms for computing various steady state performance measures and illustrative numerical examples are also solved.

References

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