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Cordial Labeling of Twin Chord

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***Abstract*: In this paper, we analyze about the product cordial labelling for some new graphs such as cycle with one chord and cycle with twin chords.**

***Keywords*: Cordial Labeling, Vertices, chord.**

1. Introduction

Graphs are discrete structure which constitutes of vertices and edges that connect these vertices. They are not concerned with their internal properties but with their inter-relationship. By a graph theory, we mean a finite undirected graph without loops or multiple edges. Vaidya and Barasara [1] introduced the edge analogue of product cordial labeling and named it as edge product cordial labeling. Further they explicits Product Cordial Labeling for some New Graphs [2]. In this paper, we analysis the cordial labeling of one chord and twin chord.

Definition: 1.1
In the product labeling, the mapping $c : V(G) \rightarrow \{0,1\}$ is called $binary\ vertex\ labelling$ of G and $f(v)$ is called the $label$ of vertex v of G under f.

The induced edge labelling $c^* : E(G) \rightarrow \{0,1\}$ is given by

$$c^*(e = uv) = \left| c(u) - c(v) \right|.$$

Let $v_c(0), v_c(1)$ be the number of vertices of G having labels 0 and respectively under f and let $e_c(0)$, $e_c(1)$ be the number of edges of G having labels 0 and 1 respectively under c^*.

Definition: 1.2
Let G be the vertex cordial labelling of graph if,

$$\left| v_c(0) - v_c(1) \right| \leq 1 \text{ and}$$
$$\left| e_c(0) - e_c(1) \right| \leq 1.$$

A graph is cordial if it admits cordial labelling.

Definition: 1.3
Let G be the vertex cordial with induced edge labelling $c^* : E(G) \rightarrow \{0,1\}$ defined by
$c^*(e = uv) = c(u)c(v)$
if $|v_c(0) - v_c(1)| \leq 1$ and $|e_c(0) - e_c(1)| \leq 1$.

A graph G is product cordial if it admits product cordial labelling.

Definition: 1.4
Let C_m be the $chord$ of cycle is an edge joining two non – adjacent vertices of cycle C_m.

Definition: 1.5
Two chords of a cycle T_m are said to be twin chords if they form a triangle with an edge of cycle C_m.

2. Mathematical formulation

Theorem: 2.1
Cycle C_m with one chord is product cordial except when n is even and the chord is joining the vertices at diameter distance.
Define, $c : V(G) \rightarrow \{0,1\}$, we consider following two cases.
Case 2: when n is odd.
Without loss of generality we assure that let the chord is between vertex (v_1, v_c) where $3 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$.

$f(v_c) = 1, \quad 1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$
$f(v_c) = 0, \quad$ otherwise

In view of above labelling pattern we have,

$v_f(0) + 1 = v_f(1) = \left\lceil \frac{m}{2} \right\rceil$
$e_f(0) = e_f(1) = \frac{m+1}{2}$

Case 2: When m is even.
Without loss of generality we assume that let the chord is between vertex (v_1, v_i) where $3 \leq i \leq \frac{m}{2} + 1$.
Subcase1: When chord is between (v_1, v_i) where $i = \frac{m}{2} + 1$.
In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to $\frac{n}{2}$ vertices out of total m vertices. The vertex with label 0 will give rise at least $\frac{m}{2} + 2$ edges with label 0 and at most $\frac{m}{2} - 1$ edges with label 1 out of total $m + 1$ edges of G. Therefore $\left| e_c(0) - e_c(1) \right| = 3$. Here the condition does not satisfied. Hence G is not product cordial.
Subcase 2: When chord is between (v_c, v_i) where $3 \leq i \leq 1$.

$f(v_c) = 1, \quad 1 \leq i \leq 1$
$f(v_c) = 0, \quad$ otherwise

In view of the above labelling pattern we have,

$V_c(0) = v_c(1) = 1$
$e_c(0) = e_c(1) - 1 = \frac{n}{2} + 1$

Thus in every case we

$$|v_c(0) - v_c(1)| \leq 1 \text{ and } |e_c(0) - e_c(1)| \leq 1.$$

Hence the product cordial cycle C_m with one chord.

Theorem: 2.2
Cycle C_7 with chords is product cordial except when t is even and one of the chord joining vertices at diameter distance.
Proof:
Let t be the cycle graph with twin chords. Let $t_1, t_2, t_3, \dots, t_m$ be the vertices. Here graph G has t vertices and $t+2$ edges.

Define $f(v) \rightarrow \{0,1\}$, we consider following two case.

Case 1: When m is odd.
In view of the above labelling pattern we have,

$$v_c(0) + 1 = v_c(1) = \left\lceil \frac{m}{2} \right\rceil$$
$$e_c(0) = e_c(1) + 1 = \left\lceil \frac{m}{2} \right\rceil + 1 ;$$

When chords are between

(v_1, v_c) and (v_1, v_{c+1}) where $i = \left\lceil \frac{m}{2} \right\rceil$.

$$e_c(0) + 1 = e_c(1) = \left\lceil \frac{m}{2} \right\rceil + 1:$$

When chords are between

(v_1, v_c) and (v_1, v_{c+1}) where $3 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$.

Case 2: When m is even.
Suppose, when the chords are between vertex (v_1, v_c) and (v_1, v_{c+1}) where $3 \leq i \leq \frac{m}{2}$.
Subcase1: When chords are between (v_1, v_c) and (v_1, v_{c+1}) where $c = \frac{m}{2}$.

In order to satisfy the vertex condition for product cordial graph it is essential to assign label 0 to $\frac{n}{2}$ vertices out of total n vertices. The vertex with label 0 will give rise at least $\frac{m}{2} + 2$ edges with label 0 and at most $\frac{n}{2}$ edges with label 1 out of total $m + 2$ edges of G. Therefore $|e_c(0) - e_c(1)| = 2$. Thus the edge condition for product cordial graph is violated. Hence G is not product cordial.
Subcase 2:
When chords are between (v_1, v_c) and (v_1, v_{c+1}) where $3 \leq c \leq \frac{n}{2}$.

$$f(v_c) = 1, \quad 1 \leq c \leq \frac{m}{2}$$
$$f(v_c) = 0, \quad \text{otherwise}$$

In view of the above labelling pattern we have,

$$v_c(0) = v_c(1) = \frac{m}{2}$$
$$e_c(0) = e_c(1) = \frac{m}{2} + 1$$

Thus in each cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence cycle S with twin chords is product cordial.

3. Conclusion

Labeling of separate structure could be a potential space of analysis. We have discussed the total edge product cordial labeling for rectilinear related graph and derive several results on it.

References

[1] S. K. Vaidya and N. B. Vyas, "Product Cordial Labeling in the context of Tersor Product of Graphs," *Journal of mathematics Research*, vol. 3, no. 3, pp. 83 – 88, 2011.
[2] S. K. Vaidya and C. M. Barasara, "Product Cordial Labeling for some New Graphs, *Journal of mathematics Research*, vol. 3, no. 2, pp. 206 – 211, 2011.