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Domination Number of Jump Graphs

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***Abstract*: The graph theory properties of $\gamma'[J(G)]$ and its exact values for some standard graphs. The relation between $\gamma'[J(G)]$ with other parameter are also discussed. We also present some results domination graph $M_{ed}(J(G))$ for some graphs.**

***Keywords*: domination number, jump graph.**

1. Introduction

Over the past decay, the domination number of jump graph is one of the new concepts in graph theory which has attracted several researchers because of various applications such as linear algebra and optimization, communication networks, social sciences and in the existing literature. The concept of edge domination was introduced by Mitchell and Hedetniemi [1] and it was explored by many researchers. Recently Arumugam and Velammal have discussed the edge domination in graphs [2]. Herein, we investigate domination number in jump graph.

2. Mathematic formulation

Definition: 1.1

The line graph $L(G)$ of G have the edges of G and vertices be adjacent in $L(G)$. Let the complement of line graph $L(G)$ as the jump graph $J(G)$ of G. The jump graph $J(G)$ is defined on $E(G)$ whose vertices are adjacent. Since both $L(G)$ and $J(G)$ are defined on the edge set of a graph G.

Definition: 1.2

The set $A \subseteq E[J(G)]$ be the dominating set of $J(G)$, if every edge not in A is adjacent to a edge in D. The dominal number of jump graph, denoted by $\gamma'[J(G)]$, is the minimum cardinality of a dominating set in $J(G)$.

Remark: 1.3

For any graph G with $p \leq 4$, the jump graph $J(G)$ of G, is disconnected since we study only the connected jump graph, we choose $p > 4$

Theorem: 1.4

1. For any path P_p, with $p \geq 5$, $\gamma'\left[J\left(P_p\right)\right] = 2$.
2. For any Cycle C_p, with $p \geq 5$, $\gamma'\left[J\left(C_p\right)\right] = 2$
3. For any Complete graph, K_p with ≥ 5 , $\gamma'\left[J\left(K_p\right)\right] = 3$.
4. For any complete bipartite graph K_{mn},
$$\gamma'[J(K_{mn})] = \begin{cases} 2 \ for \ K_{2,n} & where \ n > 2 \\ 3 \ for \ K_{m,n} & where \ m,n \geq 3 \end{cases}$$
5. For any wheel W_p, $\gamma'\left[J\left(W_p\right)\right] = \begin{cases} 3 \ for \ p = 5,6 \\ 2 \ for \ p \geq 7 \end{cases}$

Theorem: 1.5

For any connected graph G $\gamma'[J(G)] \geq 2$

Proof of the theorem is obvious

The possible theorem gives the relationship between edge domination number of a graph with its edge domination number of a jump graph.

Theorem: 1.6

For any connected graph G with diameter, $diam(G) \geq 2$, $\gamma'[J(G)] \geq 2$

Proof:

Let a, b be a path of maximum distance in G. Then $d(a,b) = diam(G)$.

We can prove the theorem with the following cases.

Case (1):

For $diam(G) = 2$, Choose a vertex v_1 of eccentricity 2 with maximum degree among others. Let $E_1 = \{e_1^1, e_2^1, \dots\}$ corresponding to the elements of $\{v_1, v_2, \dots\}$ forming a dominating set in jump graph $J(G)$. Every edge $v \notin E_1$ is adjacent to a edge in E_1. Hence E_1 is a minimum dominating set. the jump graph is $\gamma'[J(G)] > 2$.

Case (2):

For $diam(G) > 2$,

let v_1 be any vertex adjacent to v and v_2 be any vertex adjacent to u.

Let $\{v_1, v_2\} \subseteq \mathrm{V}(G)$ form a corresponding edge se $\{e_1^1, e_2^1\} \subseteq E\big(J(G)\big)$.

Since these edges $\{e_1^1, e_2^1\}$ are adjacent to all other edges of $E\big(J(G)\big)$, it follows that $\{v_1, v_2\}$ becomes a minimum dominating set. Hence, $\gamma'[J(G)] = 2$.

From this, we show G $\gamma'[J(G)] \geq 2$.

Theorem: 1.7

For any tree T with diameter greater than 3, $\gamma'[J(T)] = n$.

Proof:

When diameter of the jump graph is less than 3, then it will be dislocated.

Let uv be a path of maximum length in a tree T where diameter is greater than 3. Let e_i be the pendent vertex adjacent to u and e_k be the pendent edge adjacent to v. The edge set $e_i, i = 1,2,3, \dots n$, of $J(T)$ corresponding to the vertices in T will form the dominating set in $J(T)$. Since all the other edges of $E[J(T)]$ are adjacent with $e_i, i = 1,2,3, \dots n$ it form a minimum dominating set.

Hence $\gamma'[J(T)] = n$.

Theorem: 1.8

For any connected (p,q) graph G, $\gamma'[J(G)] \leq p - \Delta(G)$ where $\Delta(G)$ is the maximum degree of G.

Proof:

Let the set of edges be $E = \{e_1, e_2, \dots, e_n\}$ in G and $E_1 = E - e$ where e_1 is one of the edge with maximum degree. By definition of jump graph, $E(G) = V[J(G)]$. Consider $I = \{v_1, v_2, \dots, v_n\}$ as the set of vertices adjacent to e_1 in G. Let $H \subseteq E[J(G)]$ be the set of edges of $J(G)$ such that $H \subseteq V - I$. Then H itself forms a minimally dominating set. Therefore $\gamma'[J(G)] \leq |V| - |I|$.

Hence $\gamma'[J(G)] \leq p - \Delta(G)$.

Theorem: 1.9

For any connected (p,q) graph G, $2 \leq \gamma'[J(G)] \leq \lfloor p/2 \rfloor$

Proof:

An edge $\{e_i\}$ is any connected graph G is adjacent to atleast one more edge in G. In jump graph, the vertex $\{e_1'\}$ corresponding to $\{v_i\}$ is non adjacent to $\{e_i{}^k, e_j{}^j\}$ of $\{v_k, v_j\}$ in $J(G)$. Therefore by definition of edge dominating number of graph $\gamma'[J(G)]$, the dominating set contains atleast two elements.

Hence $\gamma'[J(G)] \geq 2 \rightarrow (1)$

Let E by the set of edges in G. Then $V[J(G)]$. Suppose $D = \{e_1, e_2, \dots, e_k\}$ be the dominating set. Then $E - D$ is also a dominating set. So by the definition of edge domination number of graph, we can say edge domination number $\gamma'[J(G)]$ of jump graph is given by $\gamma'[J(G)] \leq min\{|D|, |E = D|\} \leq \lfloor 1/2 \rfloor \rightarrow (2)$

from (1) and (2) we get $2 \leq \gamma'[J(G)] \leq \lfloor p/2 \rfloor$.

3. Conclusion

Here, we present some results on these types of graphs and also we discuss about the zero divisor graph of a lattice and properties of the zero divisor graph of a lattice.

References

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