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Cordial Labeling of Rectilinear Grid

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***Abstract*: In this paper we introduce an edge product cordial labeling graph. For a graph, G = (V(G), E(G)), a function f: E(G)→ {0 , 1} is called an edge product cordial labeling of G if the induced vertex labeling function defined by the product of incident edge such that the edges labeled with 1 and label 0 differ by at most 1.**

***Keywords*: Cordial Labeling, Vertices, edges, total edge product.**

1. Introduction

Graphs are discrete structure which constitutes of vertices and edges that connect these vertices. They are not concerned with their internal properties but with their inter-relationship. By a graph theory, we mean a finite undirected graph without loops or multiple edges. Vaidya and Barasara [1] introduced the edge analogue of product cordial labeling and named it as edge product cordial labeling. Aisha explicits the 3-total edge product cordial labeling of rhombic grid [2]. In this paper, we analysis the cordial labeling of rectlinear grid.

Definition: 1.1

The assignment of integers to the *vertices* or *edges*, or both, which was subjected to certain condition(s) is called graph labeling. If the domain of mapping is the set of vertices (or edges) then the labeling is called a *vertex(or an edge)* labeling.

Definition: 1.2

For a graph G, an edge labeling function $\varphi: V(G)$ is denoted by the set {0,1} which induces by the vertex labeling function $\varphi^*: E(G)$ denotes {0,1} defined as $\varphi^*(xy) = |\ \varphi(x) - \varphi(y)$.

Let us consider the number of vertices of G labeled with i under φ be $v_\varphi(i)$ and the number of edges G labeled under φ^* be $e_\varphi(i)$ for i = 0,1. This function φ is said to be *cordial labeling of* G.

Since $|e_\varphi(0) - e_\varphi(1)| \leq 1$ and $|v_\varphi(0) - v_\varphi(1)| \leq 1$. A graph is said to be cordial labeling.

Definition: 1.3

For a graph G, a vertex labeling function $\varphi: V(G) \to \{0,1\}$ induces an edge labeling function $\varphi^*: E(G) \to \{0,1\}$ defined as $\varphi^*(xy) = \varphi(x)\ \varphi(y)$. The function φ is called *total product cordial labeling* of G if $|(v_\varphi(0) + e_\varphi(0)) - (v_\varphi(1) + e_\varphi(1))| \leq 1$. A graph is called *total product cordial* if it admits *total product cordial labeling.*

Definition: 1.4

For a graph G, an edge labeling function $\varphi^*: E(G) \to \{0,1\}$ induces a vertex labeling function $\varphi: V(G) \to \{0,1\}$ defined as $\varphi(v) = \prod\{\varphi^*(xy) / xy \in E(G)\}$. The function φ^* is called a *total edge product cordial labeling* of G,

if $|(v_\varphi(0) + e_\varphi(0)) - (v_\varphi(1) + e_\varphi(1)| \leq 1$.

A graph is said to be *total edge product cordial* if it admits *total edge product cordial labeling.*

Definition: 1.5

Let us consider $C_m^{(t)}$ denote the one – point union of t *cycles* of length m.

Definition: 1.6

The *Rectilinear* R_m is defined to be the join $C_m + K_1$. The vertex corresponding to K_1 is known as *apex vertex,* the vertices corresponding to cycle are known as *rim vertices.*

2. Mathematical formulation

Theorem: 2.1

The rectilinear grid graph with degree sequences (1,1), (2,2,2,2) or (3,2,2,1) are not total edge product cordial graphs.

Proof:

For the graph with degree sequence (1, 1) has only one edge and two vertices. If we label the edge with 1 or 0 then both the vertices will give the same label.

Theorem: 2.3

The rectilnear cycle D_m is a total edge product cordial graph except for $m \neq 4$.

Proof:

Let $v_1, v_2, v_3, v_4, \ldots\ldots\ldots v_m$ be the vertices of cycle D_m.
The following two cases.

Case 1: When m is odd.

$\varphi(v_i v_{i+1}) = 0; \quad 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor$

$\varphi(v_i v_{i+1}) = 1; \quad \left\lceil \frac{m}{2} \right\rceil \leq i \leq m - 1$

$\varphi(v_1 v_m) = 1.$

Case 2: When m is even and m ∤ 4.

$\varphi(v_i v_{i+1}) = 0; \quad 1 \leq i \leq \frac{m-4}{2}$

$\varphi(v_i v_{i+1}) = 1; \quad i = \frac{m-2}{2}$

$\varphi(v_i v_{i+1}) = 0; \quad i = \frac{m}{2}$

$\varphi(v_i v_{i+1}) = 1; \quad \frac{m}{2} + 1 \leq i \leq m\text{-}1$

$\varphi(v_1 v_m) = 1.$

In both the cases we have $v_f(0) + e_f(0) = m$ and
$v_\varphi(1) + e_\varphi(1) = m.$

So, $|(v_\varphi(0) + e_\varphi(0)) - (v_\varphi(1) - e_\varphi(1))| \leq 1$. Hence, the diamond cycle D_m is a total edge product cordial graph except for $m \neq 4$.

Example 2.2

The rectlinear graph $D_4^{(3)}$ and its total edge product cordial labeling.

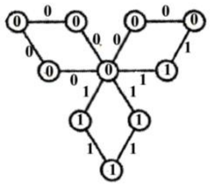

Fig. 1. Rectilinear graph

Theorem: 2.3

The rectlinear shape D_m is a total edge product cordial graph.

Proof.

Let $v_1, v_2, v_3, \ldots\ldots\ldots\ldots\ldots, v_m$ be the rim vertices and v be an apex vertex of wheel D_m. To define $\varphi: E(D_m)$ denotes the set {0,1} we will consider following two cases.

Case 1: When m is even.

$\varphi\ (v_{2i-1}v_{2i}) = 0; \quad 1 \le i \le \frac{m}{2}$

$\varphi\ (v_1v_n) = 1;$

$\varphi\ (v_{2i}v_{2i+1}) = 1; \quad 1 \le i \le \frac{m-2}{2}$

$\varphi\ (vv_i) = 1; \quad 1 \le i \le m.$

We have

$v_\varphi(0) + e_\varphi(0) = \frac{3m}{2}$ and $v_\varphi(1) + e_\varphi(1) = \frac{3m}{2}+1.$

Case 2: When m is odd.

$\varphi(v_{2i-1}v_{2i}) = 0; \quad 1 \le i \le \frac{m-1}{2}$

$\varphi(v_1v_m) = 0;$

$\varphi(v_{2i}v_{2i+1}) = 1; \quad 1 \le i \le \frac{m-1}{2}$

$\varphi(vv_i) = 1; \quad 1 \le i \le m.$

We have $v_\varphi(0)+e_\varphi(0) = \frac{3m+1}{2}$ and

$e_\varphi(1)+e_\varphi(1) = \frac{3m+1}{2}$

For both the cases, we have

$$\left| (v_\varphi(0)+e_\varphi(0)) - (v_\varphi(1)+e_\varphi(1)) \right| \le 1.$$

Hence, the rectilinear grid D_m is a total edge product cordial graph.

3. Conclusion

Labeling of separate structure could be a potential space of analysis. We have discussed the total edge product cordial labeling for rectilinear related graph and derive several results on it.

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