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Cordially Divisor Parallelogram Graphs

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***Abstract*: In this paper, we introduce some parallelogram divisor cordial graphs. Further, we can prove that the graphs obtained by the identification of some vertices of a divisor cordial graphs to certain album graphs are divisor cordial.**

***Keywords*: Divisor Cordial graph, Vertices, edges, album.**

1. Introduction

Number Theory is a fascinating subject in mathematics. It has so many interesting concepts. The concepts of primality and divisibility play an important role in Number Theory [1].
Definition 1.

Let $G = (V, E)$ be the function of $f{:}v$ is denoted by the set $\{0,1\}$ with an each edge xy, is ascribed by the label 1 if f(x) divides f(y) or f(y) divides f(x) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

For each edge xy, assign the label 1 if either $[f\ (x)]^2|f\ (y)$ or $[f\ (y)]^2|f\ (x)$ and the label 0 otherwise. f is called a rectangular divisor cordial labeling if $\left|e_f(0) - e_f(1)\right| \leq 1$. A graph with a parallelogram square divisor cordial labeling is called a parallelogram divisor cordial graph.

Definition: 1.1
One edge union of cycles having same length is called a notebook. By common, the edge is said to be the base of the notebook. If we assume t copies of cycles of length m then the notebook is denoted by $N_m^{(t}$. Note that $N_m^{(t)}$ has $(m-2)$t + 2 vertices and $(m-1)+1$ edges.

2. Mathematical formulation

Theorem: 2.1
A notebook N with parallelogram pages is divisor cordial.
Proof:
Let N be the notebook with parallelogram pages. Note that it has 2t +2 vertices and 3t +1 edges. Label the vertices of common edge by 1 and 2. Then label the vertices of the edges which are parallel to common edge as given below.
Example: 2.2
Let us consider the notebook N with 2 parallelogram pages. Note that it has 6 vertices and 7 edges.
Here, we have $e_f(0) = 3$ and
$e_f(1) = 4$.

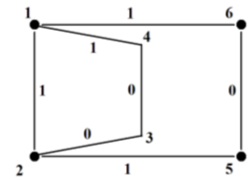

Fig. 1. Album has 2 parallelogram pages

In the first page, label the numbers 4 and 3, second page 5 and 6. Since 1 divides all the integers it contributes
t+1 to $e_f(1)$, 2 divides all the even integers it contributes $\frac{t}{2}$ to each $e_f(0)$ and $e_f(1)$.
When t is even,

$$e_f(0)= \frac{t+1}{2} \text{ and}$$
$$e_f(1)= \frac{t-1}{2}.$$

When t is odd, $m \neq m+1$ for any integer $m > 1$, the parallel edges are assigned t to $e_f(0)$.
Consequently,
Case (1) if t is even,

$$e_f(0)= \frac{3t}{2} \text{and}$$
$$e_f(1)= \frac{3t}{2} + 1 \text{ and}$$

Case (2) if t is odd, then

$$e_f(0) = \frac{3t+1}{2} \text{ and}$$
$$e_f(1)= \frac{3t+1}{2}$$

Thus, $|e_f(0) - e_f(1)| \leq 1$. As a consequence, N shows divisor cordial.

Corollary: 2.3
A notebook with even number of parallelogram pages is divisor dominated cordial but not strict.
Proof:
The notebook N is divisor dominated cordial graph. If we interchange the labels of second page, then N becomes non divisor dominated cordial.
Theorem: 2.4.
Let G be a divisor cordial graph and N be the album with parallelogram pages. Then G_N* N is divisor cordial.
Proof:
Let us assume G is a divisor cordial graph of order p and size q and the vertices labeled 1 and 2 are not adjacent. Here, f^* be the divisor.

Let N be a notebook with t parallelogram pages labeled at f_N. Now identify the vertices labeled 1 and 2 in G to the vertices of common edge of N. We already proved that $G_N *N$ be the divisor cordial. Let f be the labeling of $G_N *N$.

Case (i): if p is even.

Since G is divisor cordial, we have $e_{f^*}(0) = e_{f^*}(1) = \frac{t}{2}$.

Case (ii): if t is even, the vertices of the parallel edges in N to edge of the vertices are labeled 1 and 2 as follows.

If n is even,

$$e_f(0) = \frac{m}{2} + \frac{3t}{2} \text{ and}$$

$$e_f(1) = \frac{m}{2} + \frac{3t}{2} + 1.$$

If t is odd, divisor dominated cordial which implies

$$e_{f^*}(0) = \frac{m+1}{2} \text{ and}$$

$$e_{f^*}(1) = \frac{m+1}{2}$$

In all cases, $|e_f(0) - e_f(1)| \leq 1$. So, $G_N *N$.is divisor cordially

Example: 2.5

Consider the following divisor cordial graph G

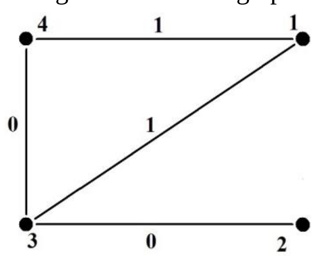

Fig. 2. Album has not adjacent

It has even order and even size. Note that the vertices labeled 1 and 2 are not adjacent.

Now, we shall connected with the album N with parallelogram pages to G shows in fig.

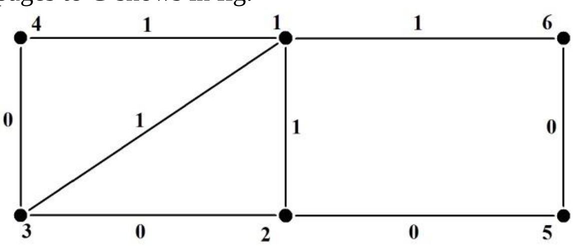

Fig. 3. Album with parallelogram pages

Here, we see that $e_f(0) = 4$ and $e_f(1) = 4$

This example illustrates the subcase(a) of Case (i) for even order of G.

Next, we shall explain the subcase (a) of Case (ii) by the following example.

Example: 2.6

Consider the following divisor cordial graph G of odd size.

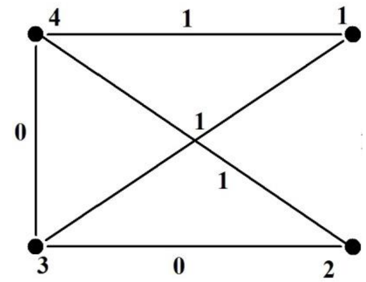

Fig. 4. Album divisor cordial graph G of odd size

Here, p = 5 and q = 4 and $e_f(0) = 2$ and $e_f(1) = 3$

Then album N is attached with parallelogram pages as given below.

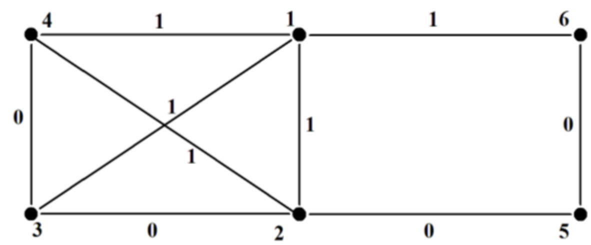

Fig. 5. Album with rectangular pages is adjacent

Here, we see that $e_f(0) = 4$ and $e_f(1) = 5$

Theorem: 2.7.

Let G be a divisor cordial graph and N be a book with the parallelogram pages and let e be the common edge of N. Then G $*$G (N − e) is divisor cordial.

Proof:

Here the vertices labeled 1 and 2 in G are adjacent.

Case (i): (a) if m is even. t is even.

Here $e_f(0) = e_f(0) = \frac{m}{2} + \frac{3t}{2}$.

(b)if m *is* even, *t* is odd

Here $e_f(0) = \frac{m}{2} + \frac{3t}{2}$ and $e_f(0) = \frac{m}{2} + \frac{3t}{2}$.

Case (ii): (a) if *m* is odd, *t* is even.

Here $|e_f(0) - e_f(1))| \leq 1$.

(b): *m* is odd, *t* is odd.

From this, we were interchanging the labels of the vertices of second page of N.

Then, we have $|e_f(0) - e_f(1)| \leq 1$.

Thus, in all the cases we see that $G *_G (N - e)$ is divisor cordial.

Example: 2.8

Consider the following divisor cordial graph G.

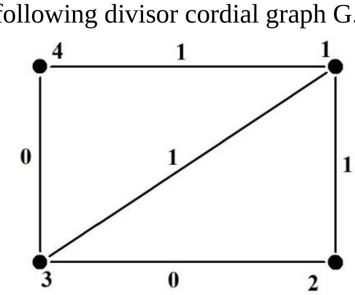

Fig. 6. Album divisor cordial graph G of even or odd size

Here, $e_f(0)$ = 2 and $e_f(1)$= 3. It is of even order and odd size. Note that the vertices labeled 1 and 2 are adjacent.
However, we shall attach N− e with 3 parallelogram pages to G as given below.

Here, we see that $e_f(0)$ = 4 and $e_f(1)$= 4

3. Conclusion

The album of the parallelogram divisor cordial graphs is discussed and also, we can prove that the graphs obtained by the identification of some vertices of a divisor cordial graphs to certain notebook graphs are divisor cordial.

References

[1] D. M. Burton, *Elementary Number Theory*, Second Edition, Wm. C. Brown Company Publishers, 1980.