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Standard Edge Domination in Graphs

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***Abstract*: The properties of graph theory $\eta'[L(G)]$ and its exact values for some standard graphs. The relation between standard edge domination with other parameter is also discussed. We also present some results or standard edge domination graph $S_{ed}L(G)$) for some graphs.**

***Keywords*: standard, domination set, path, cycle.**

1. Introduction

Over the past decay, the domination number of graph is one of the new concepts in graph theory which has attracted several researchers because of various applications such as linear algebra and optimization, communication networks, social sciences and in the existing literature [1]. The concept of edge domination was introduced by Mitchell and Hedetniemi [2] and it was explored by many researchers. Herein, we derived minimal edge domination graph.

In graph theory, the domination set for a graph G = (V, E) is a subset D of V, (i.e) each vertex not in D is adjacent to at least one member of D. The domination variety is that the variety of vertices during a smallest dominating set for G.

2. Mathematic formulation

Definition: 2.1

A G_s is a standard edge dominated graph, if every minimal edge dominating set of G_m has the same cardinality. The dominating set of graph is given by $\eta(J(G))$, is the standard cardinality of a dominating set in $J(G)$.

Lemma: 2.2

If G_s is edge dominated graph and e_f is an edge of G_s , then there exists a minimum edge dominating set containing e_f and a minimum edge dominating set not containing e_f.

Definition: 2.3

A graph G_s is also called as class 1 or class 2 according to the equation

$$ds'(G_s) = \eta'(G_s) \text{ or } \eta'(G_s) + 1.$$

Proof:

To obtain an edge dominating set containing e_f Then D is minimal and since G_s is well-edge dominated, it is minimum. To obtain a minimum edge dominating set not containing e_f,

Theorem:2.4

If G_s is well-edge dominated, then G_s is of class 1

Proof:

In this every edge belongs to any one of the χ'-set. Therefore G_s is of class 1.

Theorem: 2.5

For cycle C_s with $s \geq 4$ vertices,

$$\eta'(C_s) = \frac{s}{3} \; for\; s \equiv 0(mod\ 3)$$
$$= \left\lceil \frac{s}{3} \right\rceil.$$

For path P_s with $s \geq 4$ vertices,

$$\eta'(P_s) = s, for\; t = 3s+1, s = 1,2,3,4 \ldots$$
$$= \frac{s}{3} \; for\; s \equiv 0(mod\ 3)$$
$$= \left\lceil \frac{s}{3} \right\rceil otherwise.$$

Theorem: 2.6

For any path P_s, with $t \geq 5$, $\eta'[(P_s)] = 2$.
For any Cycle C_s, with $t \geq 5$, $\eta'[(C_s)] = 2$
For any Complete graph K_s with $p \geq 5$, $\eta'[(K_s)] = 3$.

Theorem: 2.7

For cycle C_m,

$$\eta'(T(C_s)) \begin{cases} 2\left\lceil \frac{s-1}{3} \right\rceil & if\; s \equiv 0 \; or\; 2\; (mod\ 3) \\ 2\left\lceil \frac{s+1}{3} \right\rceil & otherwise \end{cases}$$

Proof:

Consider two sets of C_s. Let $v_1, v_2, \ldots, v_m$ be the vertices of the first set of C_s and let $u_1, u_2, \ldots, u_m$ be the vertices of the second set of C_s. Let $e_1, e_2, \ldots, e_m$ be the edges of the first set of C_s, and $e_1', e_2', \ldots, e_s'$ be the edges of the second set of C_s.

Then $|V(T(C_s))| = 2m$ and $|E(T(C_s)) = 4s - 1|$.

standard edge dominating set of $D_2(C_s)$ as follows:

$$F = \begin{cases} \{e_2, e_5, \ldots, e_{3i+2}, e_2', e_5', \ldots, e_{3i+2}'\} & if\; s \equiv 0\; or\; 2\; (mod\ 3) \\ \{e_2, e_5, \ldots, e_{3i+1}, e_2', e_5', \ldots, e_{3i+2}'\} \cup \{e_s, e_s'\}, & \end{cases}$$

where $0 \leq i \leq \lfloor (m-2)/3 \rfloor$ with
$|F| = 2\lceil (s-1)/3 \rceil$ for $s \equiv 0\; or\; 2\; (mod\ 3)$ and
$|F| = 2\lceil (s+1)/3 \rceil$ for $s \equiv 1 (mod\ 3)$.

Also, $\deg(e_f) = 6 = \deg(e_f') = \Delta'(T(C_m))$, $(1 \leq i \leq s)$ and each edge of $T(C_s)$ can dominate at most seven distinct edges of $T(C_s)$ including itself. But, at a time, every of at the most distinct edges $\lceil (s-2)/3 \rceil$of $T(s)$ will dominate seven distinct edges of $T(C_s)$ as well as itself and every of the remaining edges wiil dominate but less than six distinct edges of $T(C_s)$. Therefore, any set containing the edges less than that of F cannot be an edge dominating set of $T(C_s)$. This implies that the above edge dominating set F is of minimum cardinality.

Hence, minimal edge domination with minimum cardinality among all minimal edge dominating sets is $T(C_s)$.

Thus,

$$\eta'(T(C_s)) = \begin{cases} 2\left\lceil\frac{s-1}{3}\right\rceil & if\ s \equiv 0\ or\ 2\ (mod\ 3) \\ 2\left\lceil\frac{s+1}{3}\right\rceil & otherwise \end{cases}$$

Theorem: 2.8

For cycle C_s.

$$\eta'(T(C_s)) = \begin{cases} \left\lceil\frac{2s-1}{3}\right\rceil & if\ s \equiv 0\ or\ 1\ (mod\ 3) \\ \left\lceil\frac{2s+1}{3}\right\rceil & otherwise \end{cases}$$

Proof:

Let $v_1, v_2, \dots, v_s$ be the intercalary vertices of cycle C_s and let $u_1, u_2, \dots, u_{s-1}$ be the added vertices resembling the parameters to get the edges $e_1, e_2, \dots, e_s$ of C_s to obtain $T(C_s)$.

Thus, $|V(T(C_s)) = 2s|$ and

$|E(T(C_s)) = 4s|$.

Let the edges $f_1, f_2, \dots, f_s \in E(T(C_s))$

Where, $f_s = u_{[s/2]} v_{[s/2]}$ for odd m and

$f_s = u_{(s/2)} v_{(s/2)+1}$ for even s.

First, we construct the edge sets of $T(C_s)$ as follows:

$$F = \begin{cases} \{f_2, f_{5,\dots\dots,} f_{3i+2}\} & if\ s \equiv 0\ or\ 1\ (mod\ 3) \\ \{f_{2,} f_{5,} f_{3i+2}\} \cup \{f_{2s}\} & otherwise, \end{cases}$$

for $0 \le i \le \lceil(2s-4)/3\rceil$ with

$|F| = \lceil(2s-1)/3\rceil$, if $s \equiv 0\ or\ 1\ (mod\ 3)$ and

$|F| = \lfloor(2s+1)/3\rfloor$ if $s \equiv 2(mod\ 3)$.

The above set F is an standard edge dominating set of $T(C_s)$ because each edge in $E(T(C_s))$ is either in F or is adjacent to an edge in F. Since for any edge $e_f \in$ F, the set $F - \{e_f\}$ does not dominate the edges in $N(e_f)$ of $T(C_s)$, it follows that the above set F is an standard edge dominating set of $T(C_s)$.

Now, $\deg(f_i) = 6 = \Delta'(T(C_s))$ for $1 \le i \le 2s$. For every fringe edge of $T(C_s)$ will dominate at the most seven distinct edges of $T(C_s)$ as well as itself. But, at a time each of at most $\lfloor s/2 \rfloor$ distinct edges of $T(C_s)$ can dominate the distinct edges of $T(C_s)$ and each of the remaining edges can dominate the edges of $T(C_s)$. Therefore, any set containing the edges less than that of F cannot be an edge dominating set of $T(C_s)$. This implies that the above edge dominating set F is of standard cardinality.

Hence, the above set F is an edge dominating set with standard cardinality among all standard edge dominating sets of $T(C_s)$.

Thus,

$$\eta'(T(C_s)) = \begin{cases} \left\lceil\frac{2s-1}{3}\right\rceil & if\ s \equiv 0\ or\ 1\ (mod\ 3) \\ \left\lceil\frac{2s+1}{3}\right\rceil & \end{cases}$$

3. Conclusion

The standard domination number and the parameters of standard domination number of graphs like C_s, p_s, k_s are also dicussed.

References

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