International Journal of Research in Engineering, Science and Management
Volume-2, Issue-8, August-2019
www.ijresm.com | ISSN (Online): 2581-5792

A Numerical Example to Explore that the Graphs of a Continuous Function and its Inverse Can Intersect at Finitely Many Points Beyond the Identity Line

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***Abstract*: This paper illustrates a numerical example of a function to explore that the graphs of a continuous function and its inverse can intersect at finitely many points beyond the identity line. It is a counterexample to the conjecture that a function and its inverse can intersect only on the line of identity. It is also a counterexample to the conjecture that if the graphs of a continuous function and its inverse intersect at some point beyond the identity line, then they should intersect at infinitely many points.**

***Keywords*: Function, Continuous function, Inverse function, Graph of a function, Point of intersection, Counterexample**

1. Introduction

'Function and its inverse' is one of the most interesting topics to many of the mathematicians for many decades. More they learn, more they realize that it is still not sufficiently explored. Needless to say, many properties associated to function are in the form of conjecture and good numerical examples are needed to prove or disprove each of those.

We know that a function (y in terms of x) is invertible if it is one-one (injective) and onto (surjective), i.e., bijective. Let us consider the function $f: X \to Y$ $([1/2, \infty) \to [3/4, \infty))$ such that $f(x) = x^2 - x + 1$. As this function is bijective so it is invertible and its inverse is given by $f^{-1}: X \to Y$ $([3/4, \infty) \to [1/2, \infty))$ such that $f^{-1}(x) = 1/2 + \sqrt{x - 3/4}$.

When we plot the graphs (Fig.1) of f and f^{-1}, we get to know that they intersect at a point $(1, 1)$ on the identity line, i.e., on the line $y = x$. This is one of the millions of numerical examples in support of the fact that the graph of the union of f and f^{-1} is symmetric about the line of identity.

It develops a conjecture that a function f and its inverse f^{-1} can intersect only on the line of identity and nowhere else.

With time, many numerical examples have come into existence to illustrate that it is possible to get the point of intersection of the graphs of a function and its inverse somewhere not on the line $y = x$. However, in those numerical examples, either the functions are not continuous or even if so then points of intersections are infinite.

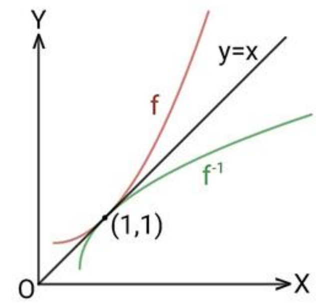

Fig. 1

The graphs of the function $f \in \{(0, 3), (1, 2), (2, 1)\}$ and its inverse $f^{-1} \in \{(1, 2), (2, 1), (3, 0)\}$ intersect at two points $(1, 2)$ and $(2, 1)$ which are not on the line $y = x$. In this case, there is no difficulty to understand that the function is discrete in nature and therefore it is not continuous. So, this function doesn't fit for the purpose.

The graphs of the function $f: X \to Y$ $([1, 3] \to [0, 2])$ such that $f(x) = 3 - x$ and its inverse $f^{-1}: X \to Y$ $([0, 2] \to [1, 3])$ such that $f^{-1}(x) = 3 - x$ intersect at points somewhere not on the line $y = x$. In this case, it is off course a continuous function but the points of intersections are infinite. So, this function also doesn't fit for the purpose.

It develops another conjecture that if the graphs of a continuous function and its inverse intersect at some point beyond the identity line then they should intersect at infinitely many points.

2. Proposal of a numerical example

The challenge was to develop a numerical example of an invertible function which is continuous as well and that the graphs of the function and its inverse intersect at finitely many points beyond the identity line.

Well, let's propose a numerical example of a function f which is appropriate for the purpose.

The function is:

$f: X \to Y$ $([2, 3] \to [2, 3])$ such that $f(x) = x^2 - 6x + 11$

This is a bijective function, so it is invertible. As it is a part of a polynomial function, therefore it is continuous as well.

Its inverse is:

$f^{-1}: X \to Y$ ([2, 3] → [2, 3]) such that $f^{-1}(x) = 3 - \sqrt{x-2}$

This inverse function f^{-1} can be treated as another numerical example of a function which is also appropriate for the purpose.

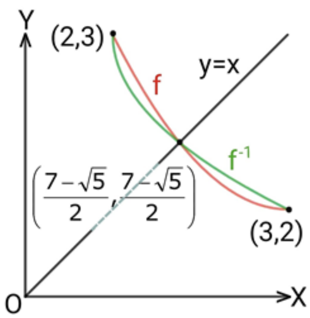

Fig. 2

When we plot the graphs (Fig. 2) of f and f^{-1}, we get to know that they intersect at three points $(2, 3)$, $\left(\frac{7-\sqrt{5}}{2}, \frac{7-\sqrt{5}}{2}\right)$ and $(3, 2)$. One can observe that only one point $\left(\frac{7-\sqrt{5}}{2}, \frac{7-\sqrt{5}}{2}\right)$ lies on the identity line. However, other two points $(2, 3)$ and $(3, 2)$ don't lie on $y = x$.

3. Conclusion

The above numerical example illustrates that the graphs of a continuous function f and its inverse f^{-1} can intersect at finitely many points beyond the identity line.

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