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Sequences and Series

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***Abstract*: The Order in which things happens. Appreciate that some of the standard series are valid for all real value of x, while others are only valid for specified ranges of value of x. Able to determine general terms in these standard cases. Able to use the method of difference to sum finite series, and extend its use to infinite series.**

***Keywords*: Mathematical induction, McLaurin series, Binomial Expansion, Fibonacci Series.**

1. Introduction

A sequence is a list of numbers. Any time you write numbers in a list format, you are creating a sequence.

We begin by discussing the concept of a sequence. Intuitively, a sequence is an ordered list of objects or events. For instance, the sequence of events at a crime scene is important for understanding the nature of the crime. In this course we will be interested in sequences of a more mathematical nature; mostly we will be interested in sequences of numbers, but occasionally we will find it interesting to consider sequences of points in a plane or in space, or even sequences of sets. The summation of all the numbers of the sequence is called Series.

A series is simply adding the terms in a sequence. An arithmetic series involves adding the terms of an arithmetic sequence and a geometric series involves adding the terms of a geometric sequence. These will be explored in other lessons.

A sequence is basically the collection of objects or elements, which is represented in an order. While the definition of series states it as, the sum of the sequence of terms. This is the basic difference between the series and sequence.

A. Sequences

A "sequence" (called a "progression" in British English) is an ordered list of numbers; the numbers in this ordered list are called the "elements" or the "terms" of the sequence. A sequence may be named or referred to by an upper-case letter such as "A" or "S". The terms of a sequence are usually named something like "a_i" or "a_n", with the subscripted letter "i" or "n" being the "index" or the counter. So the second term of a sequence might be named "a_2" (pronounced "a subscript two"), and "a_{12}" would designate the twelfth term.

The sequence can also be written in terms of its terms. For instance, the sequence of terms a_i, with the index running from $i = 1$ to $i = n$, can be written as: $(ai)^n i=1\{a\ \}\infty$

The sequence of terms starting with index 3 and going on forever could be written as: n $_{(n-3)}$.

When a sequence has no fixed numerical upper index, but instead "goes to infinity" ("infinity" being denoted by that sideways-eight symbol, ∞), the sequence is said to be an "infinite" sequence. Infinite sequences customarily have finite lower indices. That is, they'll start at some finite counter, like $i = 1$.

As mentioned above, a sequence A with terms a_n may also be referred to as "$\{an\}$", but contrary to what you may have learned in other contexts, this "set" is actually an ordered list, not an unordered collection of elements. (Your book may use some notation other than what I'm showing here. Unfortunately, notation doesn't yet seem to have been entirely standardized for this topic. Just try always to make sure, whatever resource you're using, that you are clear on the definitions of that resource's terms and symbols.) In a set, there is no particular order to the elements, and repeated elements are usually discarded as pointless duplicates. Thus, the following set:

{1, 2, 1, 2, 1, 2, 1, 2}

...would reduce to (and is equivalent to): {1, 2}

On the other hand, the following sequence:

$\{an\}$ = {1, 2, 1, 2, 1, 2, 1, 2} ...cannot be rearranged or "simplified" in any manner.

The terms of a sequence can be simply listed out, as shown above, or else they can be defined by a rule. Often this rule is related to the index. For instance, in the sequence A = $\{ai\}$ = $\{2i + 1\}$, the i-th term is defined by the rule "$2i + 1$", so the first few terms are:

$a1 = 2(1) + 1 = 3$ $a2 = 2(2) + 1 = 5$

$a3 = 2(3) + 1 = 7$

...and so forth. Sometimes the rule for a sequence is such that the next term in the sequence is defined in terms of the previous terms. This type of sequence is called a "recursive" sequence, and the rule is called a "recursion". The most famous recursive sequence is the Fibonacci sequence. Its recursion rule is as follows: $a1 = a2 = 1$; $an = an_{-1} + an_{-2}$ for $n \geq 3$

What this rule says is that the first two terms of the sequence are both equal to 1; then every term after the first two is found by adding the previous two terms. So the third term, $a3$, is found by adding $a3_{-1} = a2$ and $a3_{-2} = a1$. The first few terms of the Fibonacci sequence are: 1, 1, 2, 3, 5, 8, 13.

2. Mathematical induction

Mathematical induction is a mathematical proof technique. It is essentially used to prove that a property P(n) holds for every natural number n, i.e. for n = 0, 1, 2, 3, and so on. The method of induction requires two cases to be proved. The first case, called the base case (or, sometimes, the basis), proves that the property holds for the number 0. The second case, called the induction step, proves that, if the property holds for one natural number n, then it holds for the next natural number n + 1. These two steps establish the property P(n) for every natural number n = 0, 1, 2, 3, ... The base step need not begin with zero. Often it begins with the number one, and it can begin with any natural number, establishing the truth of the property for all natural numbers greater than or equal to the starting number.

A. Description

The simplest and most common form of mathematical induction infers that a statement involving a natural number n holds for all values of n. The proof consists of two steps:

1. The base case: prove that the statement holds for the first natural number n0. Usually, n0 = 0 or n0 = 1; rarely, but sometimes conveniently, the base value of n0 may be taken as a larger number, or even as a negative number (the statement only holds at and above that threshold), because these extensions do not disturb the property of being a well-ordered set.
2. The step case or inductive step: prove that for every n $\geq$ n0, if the statement holds for n, then it holds for n + 1. In other words, assume the statement holds for some arbitrary natural number n $\geq$ n0, and prove that then the statement holds for n + 1.

The hypothesis in the inductive step, that the statement holds for some n, is called the induction hypothesis or inductive hypothesis. To prove the inductive step, one assumes the induction hypothesis and then uses this assumption, involving n, to prove the statement for n + 1.

Whether n = 0 or n = 1 is taken as the standard base case depends on the preferred definition of the natural numbers. In the fields of combinatorics and mathematical logic it is common to consider 0 as a natural number.

Example:

Mathematical induction can be used to prove that the following statement, P(n), holds for all natural numbers n.

0+1+2+.....+n= [n(n+1)]/2

P(n) gives a formula for the sum of the natural numbers less than or equal to number n. The proof that P(n) is true for each natural number n proceeds as follows.

Base case: Show that the statement holds for n = 0 (taking 0 as a natural).

P(0) is easily seen to be true: 0= (0.0+1)/2

Inductive step: Show that if P(k) holds, then also P(k + 1) holds. This can be done as follows.

Assume P(k) holds (for some unspecified value of k). It must then be shown that P(k + 1) holds, that is:

(0+1+2+....+k)+(k+1) ={(k+1)[(k+1)+1]}/2

Using the induction hypothesis that P(k) holds, the left-hand side can be rewritten to:

[k(k+1)]/2+(k+1)

Algebraically:

[k(k+1)]/2+(k+1)=[k(k+1)+2(k+1)]/2
= [(k+1)(k+2)]/2
= {(k+1)[(k+1)+1]}/2

thereby showing that indeed P(k + 1) holds.

Since both the base case and the inductive step have been performed, by mathematical induction the statement P(n) holds for all natural numbers n.

3. McLaurin series

A Maclaurin series is a Taylor series expansion of a function about 0,

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \ldots.$$

Maclaurin series are named after the Scottish mathematician Colin Maclaurin.

The Maclaurin series of a function up to order may be found using Series [f, x, 0, n]. The th term of a Maclaurin series of a function can be computed in the Wolfram Language using Series Coefficient [f, x, 0, n] and is given by the inverse Z-transform.

Maclaurin series are a type of series expansion in which all terms are nonnegative integer powers of the variable. Other more general types of series include the Laurent series and the Puiseux series.

4. Binomial Expression

In elementary algebra, the binomial theorem (or binomial expansion) describes the algebraic expansion of powers of a binomial. According to the theorem, it is possible to expand the polynomial (x + y)n into a sum involving terms of the form axbyc, where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b. For example, (for n = 4),

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

The coefficient a in the term of axbyc is known as the binomial coefficient (n b) or (n c) (the two have the same value). These coefficients for varying n and b can be arranged to form Pascal's triangle. These numbers also arise in combinatorics, where (n b) gives the number of different combinations of b elements that can be chosen from an n-element set. Therefore (n b) is often pronounced as "n choose b".

Binomial Coefficient:

The coefficients that appear in the binomial expansion are called binomial coefficients. These are usually written (n k), and pronounced “n choose k”.

Series

A "series" is what you get when you add up all the terms of a sequence; the addition, and also the resulting value, are called the "sum" or the
"summation". For instance, "1, 2, 3, 4" is a sequence, with terms "1", "2", "3", and "4"; the corresponding series is the sum "1 + 2 + 3 + 4", and the value of the series is 10. An arithmetic series is the sum of the terms of an arithmetic sequence. A geometric series is the sum of the terms of a geometric sequence. There are other types of series, but you're unlikely to work with them much until you're in calculus. For now, you'll probably mostly work with these two. This page explains and illustrates how to work with arithmetic series. For reasons that will be explained in calculus, you can only take the "partial" sum of an arithmetic sequence. The partial sum is the sum of a limited (that is to say, a finite) number of terms, like the first ten terms, or the fifth through the hundredth Terms. If we have an arithmetic *sequence*, adding up the terms gives us an arithmetic *series*. Once we realize it’s arithmetic, and we get the rule for each term in the form,

$$a_n = a + (n-1)d$$

Then there’s an easy formula for adding up the first n terms of an arithmetic series. It’s

$$S_n = n\left(\frac{a + a_n}{2}\right)$$

Where S_n means the ‘sum of the first n terms’ (we could also use sigma notation, but this is briefer sometimes). You’ll note that a is the first term of the series, and a_n is the last term. So our formula is pretty easy, it’s just taking the average of the first and last terms, and multiplying by the number of terms.
Example: Find the sum of the series

$$\sum_{i=1}^{25} 3i + 4$$

For solution please refer below images:
Now this is the same rule as we added up in the previous example, but having 25 terms makes it very hard to do by hand. If it’s arithmetic, however we can use the formula to avoid having to write out the sequence in full. Now the first few terms of the series are 7+10+13+…. So clearly it’s arithmetic, with a = 7 and d = 3 (d, remember, is the difference between successive pairs of terms.

5. Fibonacci Series

In mathematics, the Fibonacci numbers, commonly denoted Fn form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. That is,
$F_0 = 0$, $F_1 = 1$
and
$F_n = F_{n-1} + F_{n-2}$,
For n>1,

One has $F_2 = 1$. In some books, and particularly in old ones, F_0, the "0" is omitted, and the Fibonacci sequence starts with $F_1 = F_2 = 1$.

Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are named after Italian mathematician Leonardo of Pisa, later known as Fibonacci. They appear to have first arisen as early as 200 BC in work by Pingala on enumerating possible patterns of poetry formed from syllables of two lengths. In his 1202 book Liber Abaci, Fibonacci introduced the sequence to Western European mathematics, [6] although the sequence had been described earlier in Indian mathematics.

Fibonacci numbers appear unexpectedly often in mathematics, so much so that there is an entire journal dedicated to their study, the Fibonacci Quarterly. Applications of Fibonacci numbers include computer algorithms such as the Fibonacci search technique and the Fibonacci heap data structure, and graphs called Fibonacci cubes used for interconnecting parallel and distributed systems.

6. Results

Arithmetic progression(AP) or arithmetic sequence is a sequence of numbers in which each term after the first is obtained by adding a constant, d to the preceding term. The constant d is called common difference.
In mathematics, a sequence is a list of objects (or events) which have been ordered in a sequential fashion; such that each member either comes before, or after, every other member. ...

A series is a sum of a sequence of terms. That is, a series is a list of numbers with addition operations between them.

A Taylor series is an idea used in computer science, calculus, and other kinds of higher-level mathematics. It is a series that is used to create an estimate (guess) of what a function looks like. There is also a special kind of Taylor series called a Maclaurin series.

7. Conclusion

Series are similar to sequences. Actually, the main difference between a series and a sequence is that a series is the sum of the terms of a sequence. In a series, when mathematicians talk of convergence they mean that the infinite sequence sums to a finite number.

The list of numbers written in a definite order is called a sequence. The sum of terms of an infinite sequence is called an infinite series. A sequence can be defined as a function whose domain is the set of Natural numbers. Therefore, sequence is an ordered list of numbers and series is the sum of a list of numbers.

The difference between a progression and a sequence is that a progression has a specific formula to calculate its nth term, whereas a sequence can be based on a logical rule like 'a group of prime numbers', which does not have a formula associated with it.

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