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A Study on Fuzzy Soft *Gamma*-Semiring Homomorphism

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***Abstract*: In this manuscript we study the notion of fuzzy soft Gamma-Semiring homomorphism and analyzed some properties of homomorphic image of fuzzy soft Gamma-Semiring.**

***Keywords*: Soft set, Fuzzy soft set, Fuzzy Soft Gamma-Semiring, fuzzy soft ideal, fuzzy soft Gamma-Semiring homomorphism.**

1. Introduction

The generalization of rings and distributive lattices are the best algebraic structure which was initially introduced by vandiver in 1934 but non trivial examples of semiring(S) had appeared in the studies on the theory of commutative ideals of rings by Dedekind in 19th century. Semiring is a universal algebra with two binary operations called addition and multiplication where one of them distributive over the other. If in a ring, we do away with the required of having additive inverse of each element then the resulting algebraic structure also semiring. Most of the semirings have an order structure in addition to their algebraic structure. The set of all natural numbers under usual addition and multiplication of numbers is the best example of semiring. In particular, if I is the unit interval on the real line then (I, max, min) in which 0 is the additive identity and 1 is the multiplicative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semirings lie between semigroups and rings. The study of rings shows that multiplicative structure of a ring is independent of additive structure where as in semiring multiplicative structure of a semiring is not independent of additive structure of a semiring. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring.

Semiring as the basic algebraic structure it is applied in many areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. Uncertain data in many important applications in the areas such as economics, engineering, environment, medical sciences and business management could be semiring homomorphism and we study some properties of homomorphic image of fuzzy soft *Gamma*-Semiring.

2. Preliminaries

Definition 2.1: Let U be an initial universe set and E be the set of parameters. Let P(U) denotes the power set of U. A pair (f, E) is called a soft set over U where f is a mapping given by $f: E \to P(U)$.

Definition: Let U be an initial universe set E be the set of parameters and $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over U where f is a mapping given by $f: A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition: Let $(f, A), (g, B)$ be fuzzy soft sets over U. Then (f, A) is said to be fuzzy soft subset of (g, B) denoted by $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$.

Definition: A set S together with two associative binary operations called addition and multiplication is called a Semiring.

i. Addition is a commutative operation,
ii. There exists $0 \epsilon S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$,
iii. Multiplication distributes over addition both from the left and from the right.

Definition: Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a *Gamma*-Semiring, if there exists a mapping $M \times \boldsymbol{\Gamma} \times M \to M$ written $x, \alpha, y)$ as $x\alpha y$ such that if satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \boldsymbol{\Gamma}$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$,
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Definition: let S be a *Gamma*-Semiring and A be a non-empty subset of S. A is called a *Gamma*-SubSemiring of S if A is a sub-semigroup of $(S, +)$ and $A\boldsymbol{\Gamma}A \subseteq A$.

Definition: Let S be a *Gamma*-Semiring. A subset A of S is called a left (right) ideal of S if A is closed under addition and $S\boldsymbol{\Gamma}A \subseteq A$. A is called an ideal of S if it is both a left ideal and a right ideal.

Definition: Let S be a $Gamma$-Semiring. A fuzzy subset μ of S is said to be fuzzy $Gamma$-SubSemiring of S if it satisfies the following conditions

(i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in S, \alpha \in \boldsymbol{\Gamma}$.

Example

Let M be the additive commutative semigroup of all even natural numbers with 0 and Γ be the additive semigroup of all natural numbers. Then M is a $Gamma$-Semiring if $a\gamma b$ is defined as usual multiplication of natural numbers a, γ, b where $a, b \in M, \gamma \epsilon \Gamma$. Let $A = M$ and define

$$f_a(x) = \begin{cases} 0.2\,, & if\ x = 0 \\ \dfrac{1}{2a}, & if\ x \in \{a, 2a, 3a \ldots\} \\ 0, & otherwise\,. \end{cases}$$

Let N be the additive commutative semigroup of all positive integers with 0 and Γ be the additive semigroup of all natural numbers. Then N is a $Gamma$-Semiring if $a\gamma b$ is defined as usual multiplication of integers a, γ, b where $a, b \in N, \gamma \epsilon \Gamma$. Let $B = N$ and define

$$f_a(x) = \begin{cases} 0.3\,, & if\ x = 0 \\ \dfrac{1}{b}, & if\ x \in \{b, 2b, 3b \ldots\} \\ 0, & otherwise\,. \end{cases}$$

Then (f, A) and (g, B) are fuzzy soft sets. Define $\phi: M \to N$ by $\phi(x) = x$, for all $x \in M$ and $\psi: A \to B$ by $\psi(x) = 2x$, for all $x \in A$. Then fuzzy soft function (ϕ, ψ) is a fuzzy soft $Gamma$-Semiring homomorphism from M onto N and (f, A) is soft homomorphic to (g, B).

Definition:

Let (f, A) and (g, B) be fuzzy soft $Gamma$ −Semirings over a semiring S. Then (f, A) is a fuzzy soft $Gamma$ −SubSemiring of (g, B) if it satisfies the following conditions:

(i) $A \subseteq B$,

(ii) f_a is a fuzzy $Gamma$ −Semiring of g_a for all $a \in A$.

Definition:

A fuzzy subset μ of $Gamma$-Semiring S is called a fuzzy ideal of S if it satisfies the following conditions

(i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(x\alpha y) \geq \mu(y)(\mu(x))$ for all $x, y \in S, \alpha \in \boldsymbol{\Gamma}$.

Definition:

A function $f: R \to S$ where R and S are $Gamma$ −Semirings is said to be a $Gamma$ −Semiring homomorphism if $f(a+b) = f(a) + f(b)$ and $f(a * b) = f(a) * f(b)$ for all $a, b \in R$.

Definition:

Let (ϕ, ψ) be a fuzzy soft function from R to S. The pre image of (g, B) under the fuzzy soft function (ϕ, ψ), denoted by $(\phi, \psi)^{-1}(g, B)$, defined by

$(\phi, \psi)^{-1}(g, B) = (\phi^{-1}(g), \psi^{-1}(B))$, is the fuzzy soft set.

3. Properties of fuzzy soft $GAMMA$- semiring homomorphism

In this section, the concept of fuzzy soft $Gamma$-Semiring homomorphism is analyzed and studied their properties.

Theorem

Let (f, A) be a fuzzy soft $Gamma$-Semiring over a $Gamma$-Semiring S. If $\theta: R \to S$ be an onto homomorphism and for each $a \in A$, define $(\theta f)_a(x) = f_a(\theta(x))$, for all $x \in R$, then $(\theta f, A)$ is a fuzzy soft $Gamma$-Semiring over S.

Proof:

Let $x, y \in R$, $a \in A$ and $\gamma \in \Gamma$. By the definition of $Gamma$-Sub Semiring

$$\begin{aligned} (\theta f)_a(x+y) &= f_a(\theta(x+y)) \\ &= f_a[\theta(x) + \theta(y)] \\ &\geq \min\{f_a(\theta(x)), f_a(\theta(y))\} \\ &= \min\{(\theta f)_a(x), (\theta f)_a(y)\}, \end{aligned}$$

And

$$\begin{aligned} (\theta f)_a(x\gamma y) &= f_a(\theta(x\gamma y)) \\ &= f_a[\theta(x)\gamma\theta(y)] \\ &\geq \min\{f_a(\theta(x)), f_a(\theta(y))\} \\ &= \min\{(\theta f)_a(x), (\theta f)_a(y)\}. \end{aligned}$$

Thus $(\theta f)_a$ is a fuzzy $Gamma$ −SubSemiring of S. So $(\theta f, A)$ is a fuzzy soft $Gamma$-Semiring over S.

Theorem 3.2

Let (α, A) be a fuzzy soft Semiring over $Gamma$-Semiring R. If θ is an endomorphism of R and define $(\alpha\theta)_a = \alpha_a\theta$ for each $a \in A$ then $(\alpha\theta, A)$ is a fuzzy soft $Gamma$-Semiring over $Gamma$-Semiring R.

Proof:

Let (α, A) be a fuzzy soft Semiring over $Gamma$-Semiring R. $x, y \in R$, $a \in A$ and $\gamma \in \Gamma$. By the property

$$\begin{aligned} (\alpha\theta)_a(x+y) &= \alpha_a(\theta(x+y)) \\ &= \alpha_a[\theta(x) + \theta(y)] \\ &\geq \min\{\alpha_a(\theta(x)), \alpha_a(\theta(y))\} \\ &= \min\{(\alpha\theta)_a(x), (\alpha\theta)_a(y)\}, \end{aligned}$$

and

$$\begin{aligned} (\alpha\alpha\theta)_a(x\gamma y) &= \alpha_a(\theta(x\gamma y)) \\ &= \alpha_a[\theta(x)\gamma\theta(y)] \\ &\geq \min\{\alpha_a(\theta(x)), \alpha_a(\theta(y))\} \\ &= \min\{(\alpha\theta)_a(x), (\alpha\theta)_a(y)\}. \end{aligned}$$

Thus $(\alpha\theta)_a$ is a fuzzy $Gamma$-SubSemiring of R.. So $(\alpha\theta, A)$ is a fuzzy soft $Gamma$-Semiring over S.

Theorem

Let $\phi: R \to S$ be an onto homomorphism of $Gamma$-Semirings and (α, A) be a fuzzy soft left ideal over $Gamma$-Semiring S. If for each element $a \in A, \beta_\alpha = \phi^{-1}(\alpha_a)$, then (β, A) is a fuzzy soft ideal over $Gamma$-Semiring R.

Proof:

Let $a \in A$ and $\gamma \in \Gamma$. Then α_a is a fuzzy soft left ideal over $Gamma$-Semiring S. $x, y \in R$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}\phi^{-1}(\alpha_a)(x+y) &= \alpha_a\big(\phi(x+y)\big)\\ &= \alpha_a\{\phi(x)+\theta(y)\}\\ &\geq \min\{\alpha_a\big(\phi(x)\big),\alpha_a(\phi(y))\}\\ &= \min\{\phi^{-1}(\alpha_a)(x),\phi^{-1}(\alpha_a)(y)\},\end{aligned}$$

And $$\begin{aligned}\phi^{-1}(\alpha_a)(x\gamma y) &= \alpha_a(\phi(x\gamma y))\\ &= \alpha_a[\phi(x)\gamma\phi(y)]\\ &\geq \alpha_a(\phi(y))\\ &= \phi^{-1}(\alpha_a)(y).\end{aligned}$$

Thus $\beta_\alpha = \phi^{-1}(\alpha_a)$ is a fuzzy $Gamma$-Semiring of R. So (β, A) is a fuzzy soft left ideal over $Gamma$-Semiring over R

Theorem

Let R and S be $Gamma$-Semirings, $\phi: R \to S$ be a $Gamma$-Semiring homomorphism and f be a ϕ invarient fuzzy subset of R. If $x = \phi(a)$, then $\phi(f)(x) = f(a), a \in R$.

Proof:

Let R and S be $Gamma$-Semirings, $\phi: R \to S$ be a $Gamma$-Semiring homomorphism and f be a ϕ invarient fuzzy ideal of R. Suppose $a \in R$ and $\phi(a) = x$. Then $\phi^{-1}(x) = a$.

Let $t \in \phi^{-1}(x)$. Then $\phi(t) = x = \phi(a)$. Since f is a ϕ invariant fuzzy subset of R, $f(t) = f(a)$. Thus $\phi(f)(x) = \sup_{t\in\phi^{-1}(x)} f(t) = f(a)$. So $\phi f(x) = f(a)$.

Theorem

Let (α, A) be a fuzzy soft left ideal over $Gamma$-Semiring R and ϕ be a homomorphism from R onto S. For each $c \in A, \alpha_c$ is a ϕ invarient fuzzy left ideal of R, if $\beta_c = \phi\,(\alpha_c)\; c \in A$, then (β, A) is a fuzzy soft left ideal over $Gamma$-Semiring S.

Proof:

Let $x, y \in S,\ c \in A$ and $\gamma \in \Gamma$. Then there exists $a, b \in R$ such that $\phi(a) = x, \phi(b) = y, x + y = \phi(a + b), (x\gamma y) = \phi(a\gamma b)$. Since α_c is ϕ invariant we have

$$\begin{aligned}\beta_c(x+y) &= \phi\,(\alpha_c)(x+y)\\ &= \alpha_c(a+b)\\ &\geq \min\{\alpha_c(a),\alpha_c(b)\}\\ &= \min\{\big(\phi\,(\alpha_c)(x)\big),(\phi\,(\alpha_c)(y)\}\\ &= \min\{\beta_c(x),\beta_c(y)\},\end{aligned}$$

And $$\begin{aligned}\beta_c(x\gamma y) &= \phi(\alpha_c)((x\gamma y))\\ &= \alpha_c(\phi(a\gamma b))\\ &\geq \alpha_c(\phi(b))\\ &= \phi(\alpha_c)(y)\\ &= \beta_c.\end{aligned}$$

Thus β_c is a fuzzy left ideal of S. So (β, A) is a fuzzy soft left ideal over S.

Theorem

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over $Gamma$-Semirings R and S respectively, and (ϕ, ψ) be a fuzzy soft $Gamma$-Semiring homomorphism from (f, A) onto (g, B). Then $(\phi(f), B)$ is a fuzzy soft $Gamma$-Semiring over $Gamma$-Semiring S.

Proof:

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over $Gamma$-Semirings R and S respectively, and (ϕ, ψ) be a fuzzy soft $Gamma$-Semiring homomorphism from (f, A) onto (g, B). ϕ is a $Gamma$-Semiring homomorphism from R onto S and ψ is a mapping from A onto B. For each $b \in B$, there exists $a \in A$ such that $\psi(a) = b$. Define $[\phi\,(f)]_b = \phi\,(f_a)$. Let $y_1, y_2 \in S$ and $\gamma \in \Gamma$. then there exists $x_1, x_2 \in R$ such that $\phi(x_1) = y_1, \phi\,(x_2) = y_2$ and $\phi(x_1 + x_2) = y_1 + y_2$ and $\phi\, x_1\gamma\, x_2 = y_1\gamma\, y_2$. we have the property

$$\begin{aligned}[\phi\,(f)]_{\psi(a)}(y_1+y_2) &= \phi\,(f_a)(y_1+y_2)\\ &= f_a(x_1+x_2)\\ &\geq \min\{f_a(x_1),f_a(x_2)\}\\ &= \min\{\big(\phi\,(f_a)(y_1)\big),(\phi\,(f_a)(y_2)\}\\ &= \min\{\phi\,(f)_{\psi(a)},\phi\,(f)_{\psi(a)}(y_2)\},\end{aligned}$$

And $$\begin{aligned}[\phi\,(f)]_{\psi(a)}(y_1\gamma y_2) = \phi(f_a)((y_1\gamma y_2)) &= f_a((x_1\gamma x_2)\\ &\geq \min\{f_a(x_1),f_a(x_2)\}\\ &= \min\{\big(\phi\,(f_a)(y_1)\big),(\phi\,(f_a)(y_2)\}\\ &= \min\{\phi\,(f)_{\psi(a)},\phi\,(f)_{\psi(a)}(y_2)\},\end{aligned}$$

Thus $\phi\,(f)_b$ is a fuzzy $Gamma$-Sub Semiring of S. So $(\phi\,(f), B)$ is a fuzzy soft $Gamma$-Semiring over $Gamma$-Semiring S.

Theorem

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over S. Then the following statements are true.

(i) if $g_b = f_b$ for all $b \in B \subset A$ then (g, B) is a fuzzy soft $Gamma$-SubSemiring of (f, A).

(ii) $(f, A) \cap (g, B)$ is a fuzzy soft $Gamma$-SubSemiring of (f, A) and (g, B) if it is non null.

Proof:

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over S.

(i) since $g_b \subset f_b$ for all $b \in B \subset A$, (g, B) is a fuzzy soft $Gamma$-SubSemiring of (f, A).

(ii) let $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$. Therefore (h,C) is a fuzzy soft $Gamma$-Semiring over S. Since $C = A \cup B$, (h,c) is a fuzzy soft $Gamma$-SubSemiring of (f, A) as well as (g, B).

Theorem

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over R and (f, A) be fuzzy soft $Gamma$-SubSemiring of (g, B). If $\phi: R \to S$ is a $Gamma$-Semiring homomorphism from R onto S then $(\phi\,(f), A)$ and $(\phi(g), B)$ are fuzzy soft $Gamma$-SubSemirings over S and $(\phi(f), A)$ is a fuzzy soft $Gamma$-SubSemiring of $(\phi(g), B)$.

Proof:

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over R and (f, A) be fuzzy soft $Gamma$-SubSemiring of (g, B). Since ϕ is a $Gamma$-Semiring homomorphism from R onto S, $[\phi\,(f)]_a = \phi\,(f_a)$ is a fuzzy $Gamma$-SubSemiring of S for all $a \in A$ and $[\phi\,(g)]_b = \phi\,(g_b)$ is a fuzzy $Gamma$-SubSemiring of S for all $b \in B$. then $(\phi(f), A), (\phi(g), B)$ are fuzzy soft $Gamma$-Semirings over S. By known result (f, A) is a fuzzy soft $Gamma$-SubSemiring of (g, B), f_a is fuzzy

SubSemiring of g_a. Thus $\phi(f_a)$ is a fuzzy $Gamma$-SubSemiring of $\phi(g_a)$ for all $a \in A$. So $(\phi(f), A)$ is a fuzzy soft $Gamma$-SubSemiring of $(\phi(g), B)$.

Theorem

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over R and S respectively. If (ϕ, ψ) is a fuzzy soft homomorphism from (f, A) onto (g, B), then the pre-image of (g, B) under fuzzy soft $Gamma$-Semiring homomorphism (ϕ, ψ) is a fuzzy soft $Gamma$-SubSemiring of (f, A) over R.

Proof:

Let (f, A) and (g, B) be fuzzy soft $Gamma$-Semirings over R and S respectively. Suppose (ϕ, ψ) is a fuzzy soft $Gamma$ −homomorphism from (f, A) onto (g, B).

$$(\phi, \psi)^{-1}(g, B) = (\phi)^{-1}(g), (\psi)^{-1}(B)).$$

Let $x_1, x_2 \in R$ and $\gamma \in \Gamma$. Then

$$[(\phi)^{-1}(g),]_{(a)}(x_1 + x_2) = g_{\psi(a)}[(x_1 + x_2)]$$
$$= g_{\psi(a)}[\phi(x_1) + \phi(x_2)]$$
$$\geq \min\{g_{\psi(a)}(\phi(x_1)), g_{\psi(a)}(\phi(x_2))\}$$
$$= \min\{\left((\phi)^{-1}(g)_a\right)(x_1)\big), \left((\phi)^{-1}(g)_a\right)(x_2)\}$$

And

$$[(\phi)^{-1}(g),]_{(a)}(x_1 \gamma x_2) = g_{\psi(a)}[\phi((x_1 \gamma x_2)]$$
$$\geq g_{\psi(a)}[\phi(x_1)\gamma\phi(x_2)$$
$$\geq \min\{g_{\psi(a)}(\phi(x_1)), g_{\psi(a)}(\phi(x_2))\}$$
$$= \min\{\left((\phi)^{-1}(g)_a\right)(x_1)\big), \left((\phi)^{-1}(g)_a\right)(x_2)\}$$

Thus $(\phi)^{-1}(g)$ is a fuzzy $Gamma$-SubSemiring of R.

3. Conclusion

In this study, we analyzed the notion of fuzzy soft $Gamma$-Semiring homomorphism and studied some properties of homomorphic image of fuzzy soft $Gamma$-Semiring. To extend our work, to study the anti-*Gamma* Semiring using by morphism.

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