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Common Phase Error Mitigation and Data Detection in OFDM System

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***Abstract*: The performance of multi-carrier digital communication, OFDM, undergo with the presence of phase noise. This random process is the result of the fluctuations of phases of carrier signals generated at both transmitter and receiver. Phase noise creates common phase error (CPE) that makes the signal points rotate and causes inter-carrier interference (ICI) between the sub-carriers. In this paper, K-means algorithm based CPE reparation scheme is suggested to subside the phase noise influence in OFDM system to preserve the system bandwidth. In the proposed method, firstly the data vectors in the frequency domain is clustered and then average angle of each cluster is computed. The calculated angle is compared with idle angle and the difference is measured for each cluster. Then the difference angles are averaged which is known as CPE. The performance of the suggested method is validated by a number of simulations in MATLAB and results depict that the suggested algorithm perform much better than pilot based method.**

***Keywords*: OFDM, phase noise, CPE, multi-path channel, inter-carrier interference (ICI), inter-symbol interference (ISI).**

1. Introduction

OFDM is well recognized for digital communication offered by Chang in 1966 [1]. At the time of data transmission, OFDM maintains orthogonality between the sub-channels. The main encounters in OFDM system comprise of spectral efficiency, reliability, coverage and energy efficiency for bandwidth limitation and multipath channel delay. OFDM reveals high spectral efficiency and it is vigorous against ICI and ISI. Though OFDM system has substantial benefits, it is profound to some natural impairments like frequency offset, IQ imbalance, phase noise etc. [2]-[4]. Among those, phase noise varies at every symbol that yields time-varying and random process. Therefore, the orthogonality among the sub-channel is daunted due to phase noise. Various methods are available in literature to combat the effect of phase noise [4]-[7]. Pilot based algorithm sends pilots with OFDM block to decline the effect of phase noise. In pilot based method, the use of pilot symbols lessens the bandwidth as well as reduces the throughput of the system. Moreover, pilots are needed to transmit periodically for time varying nature of phase noise. For the improvement of bandwidth, blind algorithm for phase noise mitigation have appealed much research attention around the globe. In this paper, k-means based blind phase noise mitigation technique is suggested to subside the effect of CPE.

2. Related work

To compensate for the phase noise impairment in OFDM system, in [5] authors proposed pilot based phase noise estimation method that degrade the bandwidth of the system. To increase the bandwidth efficiency, blind estimation methods are also present in the literature [9]. The conventional blind methods endure from higher computational difficulty and the performance is reduced.

3. System model

In the receiver after interleaving and mapping of information bits, frequency domain signal, X[k] is obtained that is converted to time domain signal, x[n] by using N-point IFFT. The nth sub-carrier of kth OFDM symbol can be stated in discrete-time by,

$$x[n]=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X[k]e^{j2\pi nk/N} \qquad (1)$$

Where, $k,n=0,1,\dots\dots\dots.N-1.N$ is the sub-carriers number. To avoid ISI a number of $N_c \geq L$ sub-carriers are appended at the beginning from the end of the symbol known as cyclic prefix (CP), where L is the multi-path channel length. So, the length of each transmitted OFDM block is $N_s=N+N_{cp}$.

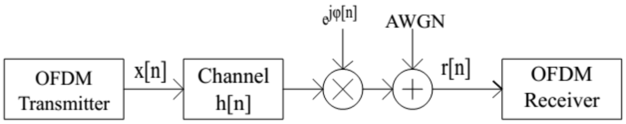

Fig. 1. Transceiver system of OFDM

After parallel to serial conversion, these N_s data are passed through digital to analog converter to obtain continuous-time signal, $x_n(t)$. By using up-conversion to higher frequency, the passband signal is given by

$$x_{nRF}=\frac{1}{\sqrt{2}}x_n(t)e^{j2\pi f_c t}+\frac{1}{\sqrt{2}}x_n^*(t)e^{-j2\pi f_c t} \qquad (2)$$

The transmitted passband signal is deformed by AWGN and phase noise. In this paper, only receiver phase noise is considered and perfect frame synchronization at the receiver is assumed.

After discarding cyclic prefix, the baseband received signal of m^{th}OFDM block in the presence of phase noise is expressed by

$$r_m = e^{j\varphi[n]} \sum_{i=0}^{L-1} h_m[i]\, x_m[n]_{modN} + w_m[n] \qquad (3)$$

where, h is channel of length L and w is AWGN noise. At the receiver N -point FFT is applied on received signal and the resulted frequency domain signal $Y_m[n]$is given by

$$\begin{aligned} R_m[n] &= \frac{1}{N}\sum_{l=0}^{N-1} r_m[l] e^{-2\pi ln/N} \\ &= \frac{1}{N}\sum_{l=0}^{N-1}\left(e^{j\varphi[l]}\sum_{p=0}^{\infty} h[p]x_m[l-p] + w[n]\right)e^{-j2\pi ln/N} \\ &= \frac{1}{N}\sum_{l=0}^{N-1}\left(e^{j\varphi[l]}\sum_{p=0}^{\infty} h[p]\left(\sum_{i=0}^{N-1} X_m(p)e^{j2\pi(l-p)/N}\right)\right)e^{-\frac{j2\pi ln}{N}} \\ &+ W[k] \end{aligned}$$

where, $W[n]$ is the FFT of AWGN noise $w[n]$, in matrix form,

$$\mathrm{R} = \mathrm{FPF^H HX+W} \qquad (4)$$

where, $\mathrm{F} = 1/\sqrt{N}\, e^{-j2\pi lm/N}, l, m = 0,1, \dots \dots \dots .. N-1$is $N \times N$ FFT matrix , $\mathrm{P} = diag(e^{j\varphi_0}, \dots \dots \dots \dots e^{\varphi_{N-1}})$ is phase noise matrix, channel matrix.
$\mathrm{H} = diag(H[0], H[1], \dots \dots .. H[N-1])$. X is data and W is AWGN with variance σ^2. Inserting the value of matrix F, P and $\mathrm{F^H}$,

$$\mathrm{FPF^H} = \frac{1}{N} \times \begin{bmatrix} \sum e^{j\varphi_n} & \sum e^{j\varphi_n} e^{\frac{j2\pi n}{N}} \dots & \sum e^{j\varphi_n} e^{\frac{j2\pi n(N-1)}{N}} \\ \sum e^{j\varphi_n} e^{-\frac{j2\pi n}{N}} & \sum e^{j\varphi_n} \dots & \sum e^{j\varphi_n} e^{\frac{j2\pi n(N-2)}{N}} \\ \vdots & \vdots & \vdots \\ \sum e^{j\varphi_n} e^{-\frac{j2\pi n(N-1)}{N}} & \sum e^{j\varphi_n} e^{-\frac{j2\pi n(N-2)}{N}} \dots .. & \sum e^{j\varphi_n} \end{bmatrix}$$

Let, $\mathrm{FPF^H} = \Phi$, then equation (4) can be expressed as

$$\begin{bmatrix} R_0 \\ R_1 \\ \vdots \\ R_{N-1} \end{bmatrix} = \frac{1}{N}\begin{bmatrix} \Phi_0 & \Phi_1 \dots & \Phi_{N-1} \\ \Phi_{N-1} & \Phi_0 \dots & \Phi_{N-2} \\ \vdots & \vdots & \vdots \\ \Phi_1 & \Phi_2 \dots .. & \Phi_0 \end{bmatrix}\begin{bmatrix} H_0X_0 \\ H_1X_1 \\ \vdots \\ H_{N-1}X_{N-1} \end{bmatrix} + \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_{N-1} \end{bmatrix}$$

So, the received signal vector R can be expressed as

$$\begin{aligned} R_m[n] &= \sum_{l=0}^{N-1} H_m[l]X_m[l]C[l-n]_N + W_m[n] \\ &= \Phi_m[0]X_mH_m \qquad (5) \\ &+ \sum_{l=0, l\neq n}^{N-1} H_m[l]X_m[l]C[l-n]_N + W_m[n] \end{aligned}$$

Phase noise angle is small and can be expressed as $e^{j\varphi[n]} = 1 + j\varphi[n]$. Therefore,

$$\begin{aligned} \Phi_m[0] &\approx \frac{1}{N}\sum_{n=0}^{N-1}(1 + j\varphi[n]) \\ &= 1 + \frac{j}{N}\sum_{n=0}^{N-1}\varphi[n] = 1 + j\bar{\varphi} \end{aligned} \qquad (6)$$

where,

$$\bar{\varphi} = \frac{1}{N}\sum_{n=0}^{N-1}\varphi[n] \qquad (7)$$

is the rotation angle due to CPE. The second term of equation (5) is ICI. The phase noise process is low and the effect of ICI is small, the equation (5) can be rewritten as

$$R_k = \Phi_0 X_k H_k + \zeta_k \qquad (8)$$

where, ζ_k comprise ICI and AWGN noise. The goal of this paper is to estimate as well as mitigate the effect of CPE.

4. Common phase error compensation

If the signal points rotate due to CPE beyond a threshold angle, the detection of received symbol will be complicated. For this reason, a CPE compensation algorithm is suggested to lessen the phase noise consequence in OFDM. Firstly, pilot based method will be clarified and after that the proposed method will be explained in details.

A. *Pilot based estimation*

Suppose, X_p is the set of pilot with locations p in an OFDM signal. By minimizing the following cost function with the help of least square method [8], the estimated CPE angle $\bar{\varphi}$ can be found as follows:

$$\min_{\Phi_0} \sum_{k\in S_p} |Y_k - \Phi_0 X_k H_k|^2 \qquad (9)$$

Differentiating with respect to Φ_0 and setting to zero yields

$$\Phi_0 = \frac{\sum_{k\in S_p} R_k (H_k X_k)^*}{\sum_{k\in S_p} |H_k X_k|^2} \qquad (10)$$

If the angle is assumed small,

$$\Phi_0 = 1 + j\varphi = \Re\left(\frac{\sum_{k\in S_p} R_k (H_k X_k)^*}{\sum_{k\in S_p} |H_k X_k|^2}\right) + j\Im\left(\frac{\sum_{k\in S_p} Y_k (H_k X_k)^*}{\sum_{k\in S_p} |H_k X_k|^2}\right) \quad (11)$$

Therefore, pilot based Least-square estimate of CPE is given by

$$\hat{\varphi}_{pilot} = \Im\left(\frac{\sum_{k\in S_p} R_k (H_k X_k)^*}{\sum_{k\in S_p} |H_k X_k|^2}\right) \quad (12)$$

B. *Proposed method*

In the proposed method, firstly the data vectors in the frequency domain is clustered and then average angle of each cluster is computed. The calculated angles are compared with idle angles and the differences are measured for each cluster. Then the difference angles are averaged which is known as common phase error (CPE). Basically, however, there are two steps to find out the CPE. For clustering of data, well known k-means algorithm is used in this paper. Two dimensional received frequency domain complex data symbol R is converted to $2xN$ data matrix S where, N is the number of data symbol and 1st and 2nd row of the matrix are for real and imaginary part of the signal respectively.

Since the algorithm is needed to be initiated with cluster agent, a $2xM$ random data vector, θ_j where, $j = 1, \dots\dots, M$,is generated where, $M = 64$ is the QAM constellation size for 64-QAM and the value of M is known to the algorithm.The function of the k-mean algorithm is to move the cluster agent to the nearest dense region of data in an iterative fashion as shown in Fig. 2. The code of the algorithm [10] is given below:

```
[d,N]=size(S);
[d,M]=size(theta);
e=1;
iteration=0;
while(e~=0)
   iteration=iteration+1;
   theta_old=theta;
   dist_all=[];
   for j=1:m
      dist=sum(((ones(N,1)*theta(:,j)'-S').^2)');
      dist_all=[dist_all; dist];
end
[q1,bel]=min(dist_all);
   J=sum(min(dist_all));
for j=1:m
      if(sum(bel==j)~=0)
         theta(:,j)=sum(X'.*((bel==j)'*ones(1,l))) / sum(bel==j);
      end
   end
   e=sum(sum(abs(theta-theta_old)));
end
```

The algorithm begin with initial value of θ_1 ,θ_2 ,…….,θ_M. After each iteration t, data S that are close to each $\theta_j(t-1)$ are identified and θ_j is updated to $\theta_j(t)$ as the average data that close to $\theta_j(t-1)$. The algorithm come to an end when no changes occur intoθ_j's between two consecutive iterations.

Above code is filed in MATLAB function kmeans.m and for iteration, the following code is used,

$$[theta, bel, J] = k\_means(S, theta\_ini)$$

Where, S is an d×N data vector matrix, *theta_ini* is a d×M matrix whose columns are the initial estimates of θ_j. *theta* is a matrix of the same size as *theta_ini*, containing the final estimates for theθ_j's, *bel* is an N-dimensional vector whose ith element contains the cluster label for the ith data vector, The k-means algorithm basically minimizes the following cost function

$$J(\theta, U) = \sum_{i=1}^{N}\sum_{j=1}^{M} u_{ij}\left\|R_i - \theta_j\right\|^2$$

Where, $\theta = [\theta_1^T, \dots, \theta_M^T]^T$, $\|\ \|$ represents Euclidian distance and $u_{ij} = 1$ if S_i close to θ_j; 0 otherwise. In words, k-means minimizes the sum of the squared Euclidean distances of each data vector from its closest parameter vector. By using the value of *bel*, the locations of all cluster and corresponding data vectors of each cluster are identified. Then, the average angle of each cluster is calculated and compared to the ideal angles.

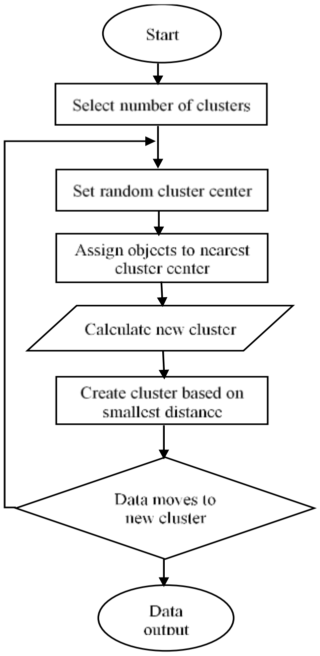

Fig. 2. Flow chart of k-mean algorithm

The difference angles are averaged again to estimate the final common phase error. The performance of the suggested method is compared with the conventional pilot based method.

5. Simulation results

To look into the performance of the projected method, the following system parameters were used to simulate the system in MATLAB environment; no. of bits, b=6, type of modulation scheme is M-QAM where, M=2b, number of sub-carrier is N=1024, 512, and 256 respectively, unloaded sub-carriers is 100, length of cyclic prefix, CP=N/4, length of Rayleigh fading channel tap, L=10, sampling frequency, fs=20 MHz, PLL bandwidth at 3dB=10 kHz and rms value of phase noise is 30. The effect of phase noise is displayed in Fig. 3. It is observed that the received data signal points are rotated from its ideal location by some angle anticlockwise (may also be clockwise) and after correction of that angles the signal points retained their original location, some noise is still available though.

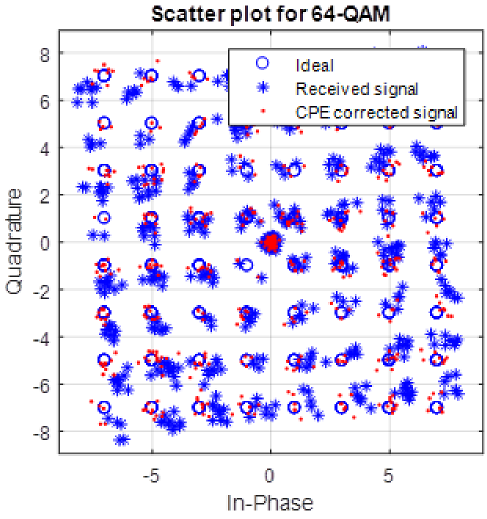

Fig. 3. Effect of phase noise in OFDM system

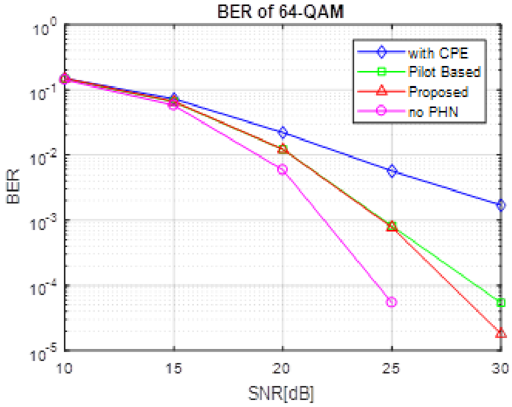

Fig. 4. BER performance for N=1024

In Fig. 4 it is noticed that the proposed method performs better in terms of bit error rate additionally saving the valuable bandwidth. In this case the number of sub-carriers were 1024. To compare the effect of number of sub-carriers on the system performance, simulation were performed by using 512 and 256 sub-carriers as shown in Fig. 5 and Fig. 6, respectively. From the figures it is clear that the performance of the proposed method outperform the convention pilot based method.

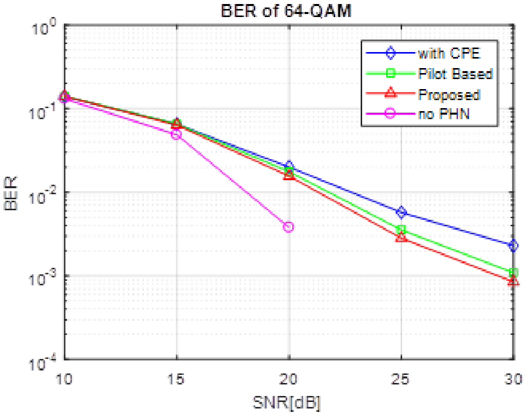

Fig. 5. BER performance for N=512

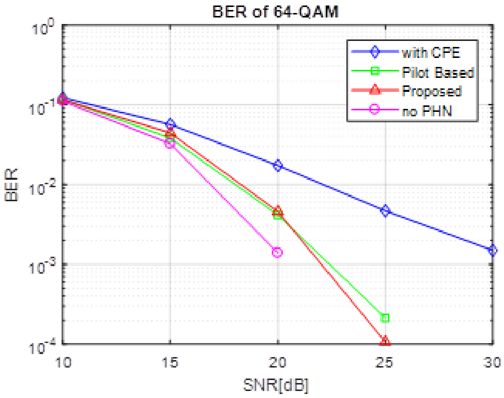

Fig. 6. BER performance for N=256

Another important fact to be mentioned that the BER decreases in reducing the number of subcarriers because the system complexity is reduced if number of subcarriers decreases. In Fig. 7 the combined performance comparison is portrayed.

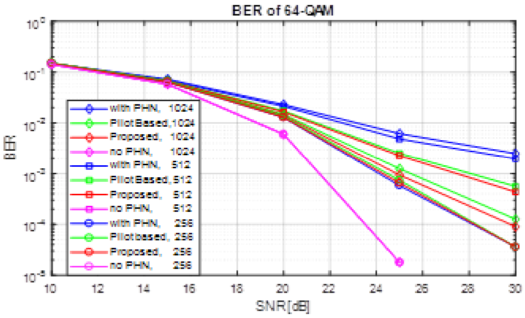

Fig. 7. BER performance comparison for N=1024, 512, and 256

6. Conclusion

In this paper a blind common phase noise estimation algorithm is proposed to protect valuable system bandwidth and to increase the throughput of OFDM system. The phase noise is estimated in frequency domain. From the simulation results it is obvious that the suggested method of estimation outperform the pilot based technique.

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