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Estimation of Shearing Force and Verification of Stokes' Law of Viscosity

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***Abstract*: A solid sphere is falling through a viscous medium under influence of gravity. Now a viscous force must act on this sphere along upward direction. It is for viscous dragging. Scientist Newton gave atfirst the idea of this force. After that scientist George Gabriel Stokes established the viscous drag formulla for a sphere when it is falling through a viscous medium. This dragging force will produce due to the addition of two forces, (1) Shearing Force and (2) Force Due To Pressure. In this paper shearing force acting on the sphere is estimated theoretically on the basis of a new concept of velocity gradient, frame of reference and equation of continuity.**

***Keywords*: cross-sectional area and Reynold's law of viscosity**

1. Introduction

We know that Scientist George Gabriel Stokes established his law in the year 1851 and now in this theorem shearing force term is derived theoretically. So it can be shown that Stokes law is valid theoretically also. It is mathematically given by

$$F = 6\pi\eta a v \text{.........} [1]$$

Here I am now going to elaborate and prove the shearing force term used on the basis of mechanical view.

A. Estimation of shearing force

When a spherical solid ball or sphere having radius "a" is allowed to fall through a liquid medium having viscosity coefficient n, it is moving along downward under the influence of gravity. So tangential force must act on the surface of that sphere. This force is given by each liquid layer to the sphere for its viscous nature. Now we devide the whole sphere into two equal hemispheres.

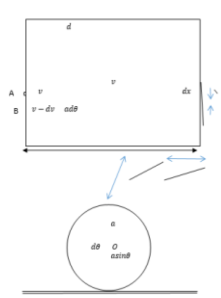

Fig. 1. A Sphere is falling through the Viscous Medium

Now for the lower hemisphere, we consider a circular elementary disc having radius $asin\theta$ and width $ad\theta$ [as shown in figure, such that the velocity of it at the layer A is v and at the layer B is $(v - dv)$, since, from the continuity equation we know for wide cross-sectional area the velocity of liquid flow must be less than that for narrow cross-sectional area, more clearly if $A1/A2 > 1$, then $V1/V2 < 1$,

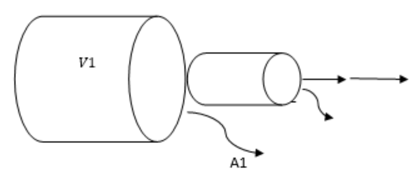

Fig. 2. Proof of Equation of Continuity

So for lower hemisphere velocity gradient must be positive, We have the velocity gradient over this layer = dv/dx, Although liquid is static and the sphere is moving with a certain velocity, but with respect to the frame of sphere, the sphere is at rest and liquid is flowing just opposite to the motion of sphere. So, effectively we can apply Reynold's law of viscosity for the elementary width dx of this liquid layer. Now it can be supposed that the liquid will pass through a region having width $(d - 2asin\theta)$ with velocity$v$, SO, We can apply here the Reynolds law related with viscosity.

Here it should be,

$$v = \frac{k\eta}{\rho(d-2asin\theta)} \text{...........} [2]$$

And hence the change of velocity between two layers be given by

$$dv = \frac{2ak\eta cos\theta}{\rho(d-2asin\theta)^2} d\theta \text{..........} [3]$$

For $dx \rightarrow ad\theta$

We get, $\frac{dv}{dx} = \frac{2k\eta cos\theta}{\rho(d-2asin\theta)^2}$ ....[4]

Where, k is Reynold's number and ρ is the density of the fluid medium, This equation [4] represents the velocity gradient of liquid. Here the tangential force will act on that layer along upward on the elementary area segment of it, and here this area must be,

$dA = 2\pi a sin\theta . a d\theta$

$= 2\pi a^2 sin\theta d\theta$ [From Fig-1]

Now, the tangential stress on that elementary segment from Newton's law of viscosity we get, Tangential stress is proportional to the velocity gradient (dv/dx),

$f \propto \frac{dv}{dx}$

$$dF = \eta . dA . \frac{dv}{dx}$$

$$\Rightarrow dF = \eta . 2\pi a^2 sin\theta d\theta \frac{2k\eta cos\theta}{\rho(d - 2asin\theta)^2}$$

$$\Rightarrow F1 = \int dF = \int_0^{\frac{\pi}{2}} \eta . 2\pi a^2 sin\theta \frac{2k\eta cos\theta}{\rho(d - 2asin\theta)^2} d\theta$$

$$\Rightarrow F1 = \frac{\pi k\eta^2}{\rho} \int_{d-2a}^{d} \frac{d - t}{t^2} dt$$

$$\Rightarrow F1 = \frac{\pi k\eta^2}{\rho} [\frac{2a}{d-2a} - \log_e \frac{d}{d-2a}] \ldots\ldots. [5]$$

This is the force acting on the sphere along upward direction at the lower hemisphere, On the other hand, for upper portion of it, the attractive force will act on the upper hemisphere along upward and the effect of this force must change layer to layer when the sphere is falling down through the viscous medium Similarly, for this case if we choose an elementary area segment on the upper hemisphere, then, here effectively we see that the velocity gradient is here actually negative, and it is (-dv/dx). The net force acting (along the upward due to adhesive attraction on the sphere by the liquid layers be given by

$$dF = \eta . dA(-\frac{dv}{dx})$$

$$\Rightarrow dF = -\eta . 2\pi a^2 sin\theta d\theta \frac{2k\eta cos\theta}{\rho(d - 2asin\theta)^2}$$

$$\Rightarrow F2 = \int dF = -\int_{\frac{\pi}{2}}^{0} \eta . 2\pi a^2 sin\theta \frac{2k\eta cos\theta}{\rho(d - 2asin\theta)^2} d\theta$$

$$\Rightarrow F2 = \int dF = \int_0^{\frac{\pi}{2}} \eta . 2\pi a^2 sin\theta \frac{2k\eta cos\theta}{\rho(d - 2asin\theta)^2} d\theta$$

$$\Rightarrow F2 = \frac{\pi k\eta^2}{\rho} \int_{d-2a}^{d} \frac{d - t}{t^2} dt$$

$$\Rightarrow F2 = \frac{\pi k\eta^2}{\rho} [\frac{2a}{d-2a} - \log_e \frac{d}{d-2a}] \ldots\ldots\ldots. [6]$$

This the effective shearing force acting on the upper hemisphere of the falling sphere.

So finally,

Total force acting on the whole hemisphere due to shearing stress acting on it i.e. total viscous drag due to shearing force becomes

$$F(shear) = F1 + F2 = \frac{2\pi k\eta^2}{\rho} [\frac{2a}{d-2a} - \log_e \frac{d}{d-2a}] \quad \ldots\ldots\ldots\ldots.. [7]$$

This is the effective shearing force acting on the whole spherical ball along upward. Now, in case if the diameter of the vessel of liquid be much greater than the diameter of the sphere, then for achieving a constant velocity (So called terminal velocity) the ball will move with velocity v. Again from the concept of frame of reference we get from Reynolds law

$$v = \frac{k\eta}{\rho(d-2asin\theta)}$$

For, $(d - 2a) \rightarrow d$ we have $(d - 2asin\theta) \rightarrow d$

Hence net shearing force become

$$F(shear) = \frac{2\pi k\eta^2}{\rho} [\frac{2a}{d}]$$

$$\Rightarrow F(shear) = 4\pi\eta a \frac{k\eta}{\rho d}$$

$$\Rightarrow F(shear) = 4\pi\eta a v \ldots\ldots\ldots [8]$$

Hence it is the actual shearing force acting on the sphere or spherical ball along upward when it falls under influence of gravity through a viscous medium. Also the force acting due to pressure on the spherical ball must be

$F(pressure) = 2\pi\eta a v$ [It was published before by U.H.Kurzweg]

So, total force becomes F =F(shear)+F(pressure) i.e.

$$F = 6\pi\eta a v$$

Hence finally the shearing force acting on the spherical ball is estimated in easy way and Stokes law of viscosity and it also supports Stokes law of viscosity. This is the estimation of shearing force by a new and very easy procedure.

2. Conclusion

This paper concludes that estimation of shearing force and verification of stokes' law of viscosity

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