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Two New Operators on Triangular Fuzzy Matrices

A. Priyanga[1], R. A. Myvizhi[2]
[1]*Assistant Professor, Dept. of Mathematics, Sri Shanmugha College of Engg. and Technology, Sankari, India*
[2]*Assistant Professor, Department of Mathematics, Kavithas College of Arts and Science, Rasipuram, India*

***Abstract*: In this paper, two new binary fuzzy operators ⊕ and ⊙ are introduced for Triangular Fuzzy Matrices (TFMs) .Using binary fuzzy operators some important properties of TFMs are presented in classical matrices .Some special types of TFMs are defined and a number of properties of these TFMs are presented.**

***Keywords*: Fuzzy matrix, Fuzzy operators, Triangular Fuzzy Matrix.**

1. Introduction

Fuzzy Matrices were introduced for the first time by Thomason ho discussed the convergence of powers of fuzzy matrix. Fuzzy Matrices play a major role in scientific development, statistics. Fuzzy Matrices arises in many applications. Several authors presented a number of results on fuzzy matrices. Fuzzy Matrices are now a very rich topic in modelling uncertain situations occurred in science, automata theory, logic of binary relations, medical diagnosis etc. Two new operators are introduced in Triangular Fuzzy Matrices (TFMs) and some properties are presented.

2. Definitions

A. Fuzzy matrix

A Fuzzy Matrix F of order $m \times n$ is defined as $F = [\langle f_{ij}, f_{ij\mu}\rangle]_{m\times n}$ where $f_{ij\mu}$ is the membership value of the element f_{ij} in F.
For simplicity ,we write F as $F = [f_{ij\mu}]$.

B. Boolean fuzzy matrix

A Fuzzy Matrix $F = [f_{ij}]_{m\times n}$ is said to be a booblean fuzzy matrix of order $m \times n$ if all the elements of F are either 0 or 1. It is obvious that (i) $1 \oplus x = x$ (ii) $1 \odot x = x$ (iii) $0 \oplus x = x$ (iv) $0 \odot x = 0$.

C. Triangular fuzzy matrix

A Triangular Fuzzy Matrix of order $m \times n$ is defined as $F = [f_{ij}]_{m\times n}$ where $f_{ij} = \langle m_{ij}, \propto_{ij}, \beta_{ij}\rangle$ is the ij^{th} element of F, m_{ij} is the mean value value of f_{ij} are the left and right spreads of f_{ij} respectively.

As for classical matrices define the following opertion on TFMs.
Let $F = (f_{ij})$ and $G = (g_{ij})$ be two TFMs of same order.
Then we have the following

1. $F + G = (f_{ij} + g_{ij})$
2. $F - G = (f_{ij} - g_{ij})$
3. For $F = (f_{ij})_{m\times n}$ and $G = (g_{ij})_{n\times p}$, $F.G = (h_{ij})_{m\times p}$
   Where $h_{ij} = \sum_{k=1}^{n} f_{ik} . g_{kj}$,$i = 1,2, \dots m$ & $j = 1,2, \dots p$.
4. $F' = (f_{ji})$ (the Transpose of F)
5. $k.F = (kf_{ij})$ where k is a scalar .

The Two new operations ⊙ and⊕ on the TFMs are defined as

$$F \oplus G = [f_{ij} + g_{ij}]$$
$$F \odot G = [f_{ij} . g_{ij}].$$

D. Symmetric TFM

A Square TFM $F = (f_{ij})$ is said to be symmetric if $F = F'$ i.e if $f_{ij} = f_{ji}$ for all i, j.

E. Pure skew symmetric TFM

A Square TFM $F = (f_{ij})$ is said to be Pure Skew Symmetric TFM
if $F = -F'$ and $f_{ii=\langle 0,0,0\rangle}$ (i.e) if $f_{ij} = -f_{ji}$ for all i ,j and $f_{ii=\langle 0,0,0\rangle}$.

F. Fuzzy skew symmetric TFM

A Square TFM $F = (f_{ij})$ is said to be Fuzzy Skew Symmetric TFM
if $F = -F'$ and $f_{ii=\langle 0,\epsilon_1,\epsilon_2\rangle}$ (i.e) if $f_{ij} = -f_{ji}$ for all i ,j and $f_{ii=\langle 0,\epsilon_1,\epsilon_2\rangle}$, $\epsilon_1, \epsilon_2 \neq 0$.

3. Basic Properties

In this section some properties of TFMs are presented.
Property 1: For any Fuzzy Triangular Matrix F,
1. $F \oplus F \geq F$

2. $F \odot F \leq F$.

Proof:

(i) The ij^{th} element of $F \oplus F$ is $(f_{ij} + f_{ij}) = 2f_{ij} \geq f_{ij}$.

(ii) The ij^{th} element $f_{ij}{}^2$ of $F \odot F$ is less than f_{ij}, Therefore $F \odot F \leq F$.

Hence $F \oplus F \geq F$ and $F \odot F \leq F$.

Property 2: Let F,G and H be any three Fuzzy Triangular Matrix,

(i) $F \oplus G = G \oplus F$

(ii) $(F \oplus G) \oplus H = F \oplus (G \oplus H)$

(iii) $F \odot G = G \odot F$

(iv) $(F \odot G) \odot H = F \odot (G \odot H)$

Proof:

(i)

Let a_{ij} and b_{ij} be the ij^{th} element of $F \oplus G$ and $G \oplus F$ respectively,

Then $a_{ij} = b_{ij}$.

Therefore $a_{ij} = f_{ij} + g_{ij}$ and $b_{ij} = g_{ij} + f_{ij}$

$= f_{ij} + g_{ij} = a_{ij}$

$b_{ij} = a_{ij}$.

Hence $F \oplus G = G \oplus F$.

(ii)

Let a_{ij} , b_{ij}, c_{ij} and d_{ij} be the ij^{th} element of $F \oplus G$, $(F \oplus G) \oplus H$, $G \oplus F$ and $F \oplus (G \oplus H)$ respectively,Then $a_{ij} = d_{ij}$.

Therefore $a_{ij} = f_{ij} + g_{ij}$, $b_{ij} = (f_{ij} + g_{ij}) + h_{ij}$ $= (f_{ij} + g_{ij} + h_{ij})$

$c_{ij} = g_{ij} + h_{ij}$, $d_{ij} = f_{ij} + (g_{ij} + h_{ij})$

$= (f_{ij} + g_{ij} + h_{ij}) = a_{ij}$

Hence $(F \oplus G) \oplus H = F \oplus (G \oplus H)$.

(i) Let a_{ij} and b_{ij} be the ij^{th} element of $F \odot G$ and $G \odot F$ respectively,

Then $a_{ij} = b_{ij}$.

Therefore $a_{ij} = f_{ij}.g_{ij}$ and $b_{ij} = g_{ij}.f_{ij}$

$= f_{ij}.g_{ij} = a_{ij}$

$b_{ij} = a_{ij}$

Hence $F \odot G = G \odot F$.

(ii) Let a_{ij} , b_{ij}, c_{ij} and d_{ij} be the ij^{th} element of $F \odot G$, $(F \odot G) \odot H$, $G \odot F$ and $F \odot (G \odot H)$ respectively,

Then $a_{ij} = d_{ij}$.

Therefore

$a_{ij} = f_{ij}.g_{ij}$, $b_{ij} = (f_{ij}.g_{ij}).h_{ij} = (f_{ij}.g_{ij}.h_{ij})$

$c_{ij} = g_{ij}.h_{ij}$, $d_{ij} = f_{ij}.(g_{ij}.h_{ij}) = (f_{ij}.g_{ij}.h_{ij})$

Hence $(F \odot G) \odot H = F \odot (G \odot H)$.

Property 3: Let F,G and H be three Triangular Fuzzy Matrices,

(i) $(F \oplus G)' = F' \oplus G'$

(ii) $(F \odot G)' = F' \odot G'$

(iii) $(F')' = F$.

Proof:

(i) Let a_{ij}, b_{ij} and c_{ij} be the ij^{th} element of $F \oplus G$, $(F \oplus G)'$ and $F' \oplus G'$respectively,

Then $b_{ij} = c_{ij}$.

Therefore $a_{ij} = f_{ij} + g_{ij}$ and $b_{ij} = f_{ji} + g_{ji}$

$c_{ij} = f_{ji} + g_{ji} = b_{ij}$

$b_{ij} = c_{ij}$

Hence $(F \oplus G)' = F' \oplus G'$

(ii) Let a_{ij}, b_{ij} and c_{ij} be the ij^{th} element of $F \odot G$, $(F \odot G)'$ and $F' \odot G'$respectively,

Then $b_{ij} = c_{ij}$.

Therefore $a_{ij} = f_{ij}.g_{ij}$ and $b_{ij} = f_{ji}.g_{ji}$

$c_{ij} = f_{ji}.g_{ji} = b_{ij}$

$b_{ij} = c_{ij}$

Hence $(F \odot G)' = F' \odot G'$.

(iii)

Let a_{ij}, b_{ij} and c_{ij} be the ij^{th} element of $F, F', (F')'$ respectively,

Then $a_{ij} = c_{ij}$

Therefore, $a_{ij} = f_{ij}$, $b_{ij} = f_{ji}{}'$, and

$c_{ij} = \left(f_{ij}{}'\right)' = f_{ji}{}' = f_{ij} = c_{ij}$

Hence $(F')' = F$.

Property 4: Let F, G and H be three Triangular Fuzzy Matrices, If $F \leq G$,then

$F \oplus \mathrm{H} \leq \mathrm{G} \oplus H$ and $F \odot \mathrm{H} \leq \mathrm{G} \odot H$.

Proof:

Let a_{ij} , b_{ij}, c_{ij} and d_{ij} be the ij^{th} element of $F \oplus H$, $G \oplus H$, $F \odot H$ and $G \odot H$ Respectively.

$a_{ij} = f_{ij} + h_{ij}$, $b_{ij} = g_{ij} + h_{ij}$

$c_{ij} = f_{ij}.h_{ij}$, $d_{ij} = g_{ij}.h_{ij}$

Since $F \leq G$,

Therefore $f_{ij} \leq g_{ij} = f_{ij} + h_{ij} \leq g_{ij} + h_{ij}$

$a_{ij} = b_{ij}$

Hence $F \oplus \mathrm{H} \leq \mathrm{G} \oplus H$.

Again $F \leq G$,

Therefore $f_{ij} \leq g_{ij} = f_{ij}.h_{ij} \leq g_{ij}.h_{ij}$

$c_{ij} = d_{ij}$

Hence $F \odot \mathrm{H} \leq \mathrm{G} \odot H$.

Property 5: Let F and G be two TFMs of the same order and k,l be two scalars then

(i) $k(lF) = (kl)F$

(ii) $k(F \oplus G) = kF \oplus kG$

(iii) $(k \oplus l)F = kF \oplus lF$

(iv) $k(F \ominus G) = kF \ominus kG$

Proof:

(i) Let a_{ij} , b_{ij}, c_{ij} and d_{ij} be the ij^{th} element of (lF), $k(lF)$, (kl) and $(kl)F$

Respectively, Then $a_{ij} = b_{ij}$

Therefore $a_{ij} = (lf_{ij})$, $b_{ij} = k(lf_{ij}) = (klf_{ij})$

$c_{ij} = (kl)$, $d_{ij} = (kl)f_{ij} = (klf_{ij}) = b_{ij}$ Hence $k(lF) = (kl)F$.

(ii) Let a_{ij} , b_{ij}, c_{ij} be the ij^{th} element of $F \oplus G, k(F \oplus G)$, and $kF \oplus kG$ respectively.

Therefore $a_{ij} = f_{ij} + g_{ij}$, $b_{ij} = k(f_{ij} + g_{ij})$

$c_{ij} = kf_{ij} + kg_{ij} = k(f_{ij} + g_{ij}) = b_{ij}$ Hence $k(F \oplus G) = kF \oplus kG$.

(iii) Let a_{ij} , b_{ij}, c_{ij} be the ij^{th} element of $k \oplus l$, $(k \oplus l)F$, and $kF \oplus lF$ respectively.

Therefore $a_{ij} = k + l$, $b_{ij} = (k + l)\, f_{ij} = kf_{ij} + lf_{ij}$

$c_{ij} = kf_{ij} + lf_{ij} = b_{ij}$ Hence $(k \oplus l)F = kF \oplus lF$.

(iv) Let a_{ij} , b_{ij}, c_{ij} be the ij^{th} element of $F \ominus G$, $k(F \ominus G)$, and $kF \ominus kG$ respectively.

Therefore $a_{ij} = f_{ij} - g_{ij}$, $b_{ij} = k(f_{ij} - g_{ij})$

$c_{ij} = kf_{ij} - kg_{ij} = k(f_{ij} - g_{ij}) = b_{ij}$ Hence $k(F \ominus G) = kF \ominus kG$.

Corollary: Let F and G be two TFMs and k,l be two scalars then

(i) $(k \odot F)' = k \odot F'$

(ii) $(k \odot F \oplus l \odot G)' = k \odot F' \oplus l \odot G'$

Property 6: If F be a TFM of order $m \times n$

(i) $F \ominus F$ is a fuzzy null TFM

(ii) $F \oplus 0 = F \ominus 0 = F$.

Proof:

(i) Let a_{ij} be the ij^{th} element of $F \ominus F$

Therefore, $a_{ij} = f_{ij} - f_{ij} = 0$

Hence $a_{ij} = 0$ is a fuzzy null TFM.

(ii) Let a_{ij}, b_{ij} be the ij^{th} element of $F \oplus 0$, $F \ominus 0$

Therefore, $a_{ij} = f_{ij} + 0 = f_{ij} - 0 = f_{ij}$.

Hence $F \oplus 0 = F \ominus 0 = F$.

Property 7: Let F be a square TFM then,

(i) $F \odot F'$ and $F' \odot F$ are both symmetric

(ii) $F \odot F'$ is symmetric

(iii) $F - F'$ is fuzzy skew- symmetric.

4. Conclusion

This paper presented two new operators on triangular fuzzy matrices.

References

[1] Amiya K. Shyamal and Madhumangal pali, "Two new operators on fuzzy matrices," 15(2001), 91-107.

[2] Amiya K. Shyamal and Madhumangal pali, "Triangular Fuzzy Mtrices, 4(2007), 75-87.

[3] D. Dubois and H. Prade, fuzzy sets and systems: theory and applications," Academic press London, 1980.

[4] A. Kandel, "Fuzzy mathematical techniques with its application," Addison-Wesley, Tokyo, 1996.

[5] J. B. Kim, "Determinate theory for fuzzy and boolean matrices," congressus Numeratium, (1988) 273-276.

[6] M. Pal, "Intuitionistic fuzzy determinant," V. U. J, Physical Sciences, 7 (2001) 87-93.

[7] M. Pal, "Fuzzy matrices ith fuzzy rows and fuzzy columns," Journal of Intelligent and Fuzzy systems, 30(1), (2015), 561-573.