

# Radiation Effect on Convective Heat Transfer in a Vertical Wavy Channel with Hall Effects

R. Suresh Babu<sup>1</sup>, M. Ravindra<sup>2</sup>

<sup>1</sup>Research Scholar, Department of Mathematics, Rayalaseema University, Kurnool, India

<sup>2</sup>Reader, Department of Mathematics, S. S. B. N Degree & P.G.College, Anantapur, India

**Abstract:** We investigate the convective study of heat transfer flow of a viscous electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat generating sources. The walls of the channels are maintained at constant temperatures. The equations governing the flow and heat are solved by employing perturbation technique with a slope  $\phi$  of the wavy wall. The velocity and temperature distributions are investigated for a different Parameters. The rate of heat transfer are numerically evaluated for a different variations of the governing parameters.

**Keywords:** heat transfer, radiation effect, hall effect

## 1. Introduction

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. The study of buoyancy driven convection flows through a porous media has been stimulated by its applications in several geophysical and engineering problems. The two main configurations in which the heat transfer driven flow in a porous medium. This convection heat transfer potential flow through a porous medium is rapidly growing as an independent branch in Fluid Mechanics and Heat Transfer. This problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors in the recent times, notably Lai F.C. [11], [12], Angirasa et al [3] Abdul [2]. Natural convection in differentially heated vertical enclosures is of fundamental interest to many practical applications. Several investigators have presented analytical and experimental results on convection in the rectangular cavity with vertical walls at constant temperatures, the horizontal walls being insulated [6], [12]. Reviewed the extensive work and mentioned about [12] who have contributed to the forced convection with heat generating source. AbdEl – Naby et al [1] studied the effects of radiation on unsteady free convective flow pasta semi-infinite vertical plate with variable surface temperature using Crank – Nicolson finite difference method. Chamkha et al. [7] analyzed the effects of radiation on free convection flow past a semi-infinite vertical plate with mass transfer, by taking into account the buoyancy ratio parameter  $N$  Ganesan and Loganadhan [9] studied the radiation and mass transfer effects flow of incompressible viscous fluid past a moving vertical cylinder

using Rosseland approximation by the Crank – Nicolson finite difference method. Takhar et al. [15] considered the effects of radiation on MHD free convection flow of a radiating gas past a semi-infinite vertical plate.

Theoretical study of free convection in a horizontal porous annulus, including possible three dimensional and transient effects. Similar studies for fluid filled annuli are available in the literature [12]. In view of this, several authors, notably Tunc et. al. [16], Oliveira et al [18]. Martin ostoja [14], El – Hakein [8], and Bulent Yesilata [6] have studied the effect of viscous dissipation on convective flows past an infinite vertical plates and through vertical channels and ducts.

The application of electromagnetic fields in controlling the heat transfer as in aerodynamic heating leads to the study of magneto hydrodynamics heat transfer. The MHD heat transfer has gained significance owing to advancement of space technology. The MHD heat transfer can be divided into sections. One contains problems in which the heating is an incidental by product of the electromagnetic fields as in the MHD generators and pumps etc. and the second contains of problems in which the primary use of electromagnetic fields is to control the heat transfer. With the fuel crisis deepening all over the world there is great concern to utilize the enormous power beneath the earth's crust in the geothermal region. Liquid in the geothermal region is an electrically conducting liquid because of high temperature.

## 2. Formulation and solution of the problem

We consider the steady flow of an incompressible, viscous, electrically conducting fluid confined in a vertical channel bounded by two wavy walls under the influence of an inclined magnetic field of intensity  $H_0$  lying in the plane  $(y-z)$ . The magnetic field is inclined at an angle  $\alpha$  to the axial direction  $k$  and hence its components are  $(0, H_0 \sin(\alpha), H_0 \cos(\alpha))$ . In view of the waviness of the wall the velocity field has components  $(u, 0, w)$  The magnetic field in the presence of fluid flow induces the current  $(J_x, 0, J_z)$ . We choose a rectangular cartesian co-ordinate system  $O(x, y, z)$  with  $z$ -axis in the vertical direction and the walls at  $x = \pm f\left(\frac{\delta z}{L}\right)$ .

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\bar{J} + \omega_e \tau_e \bar{J} \times \bar{H} = \sigma (\bar{E} + \mu_e \bar{q} \times \bar{H}) \quad (1)$$

where  $\bar{q}$  is the velocity vector.  $\bar{H}$  is the magnetic field intensity vector.  $\bar{E}$  is the electric field,  $\bar{J}$  is the current density vector  $\omega_e$  is the cyclotron frequency,  $\tau_e$  is the electron collision time,  $\sigma$  is the fluid conductivity and  $\mu_e$  is the magnetic permeability. Neglecting the electron pressure gradient, ion-slip and thermo-electric effects and assuming the electric field  $E=0$ , equation (6) reduces

$$j_x - m H_0 J_z \sin(\alpha) = -\sigma \mu_e H_0 w \sin(\alpha) \quad (2)$$

$$J_z + m H_0 J_x \sin(\alpha) = \sigma \mu_e H_0 u \sin(\alpha) \quad (3)$$

where  $m = \omega_e \tau_e$  is the Hall parameter.

On solving equations (2.2)&(2.3) we obtain

$$j_x = \frac{\sigma \mu_e H_0 \sin(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (m H_0 \sin(\alpha) - w) \quad (4)$$

$$j_z = \frac{\sigma \mu_e H_0 \sin(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (u + m H_0 w \sin(\alpha)) \quad (5)$$

where  $u, w$  are the velocity components along  $x$  and  $z$  directions respectively,

The Momentum equations are

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + \mu_e (-H_0 J_z \sin(\alpha)) \quad (6)$$

$$u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + \mu_e (H_0 J_x \sin(\alpha)) \quad (7)$$

Substituting  $J_x$  and  $J_z$  from equations (4) & (5) in equations (6) & (7) we obtain

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma \mu_e H_0^2 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (u + m H_0 w \sin(\alpha)) \quad (8)$$

$$u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\sigma \mu_e H_0^2 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (w - m H_0 u \sin(\alpha)) - \rho g \quad (9)$$

$$u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\sigma \mu_e H_0^2 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (w - m H_0 u \sin(\alpha)) - \rho g$$

$$\frac{\sigma \mu_e H_0^2 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (w - m H_0 u \sin(\alpha)) - \rho g$$

The energy equation is

$$\rho C_p \left( u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} \right) = k_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q(T_2 - T) - \frac{\partial(q_R)}{\partial x} \quad (10)$$

$$+Q(T_2 - T) - \frac{\partial(q_R)}{\partial x}$$

The equation of state is

$$\rho - \rho_0 = -\beta(T - T_0) \quad (11)$$

Where  $T$  is the temperature and concentration in the fluid.  $k_f$  is the thermal conductivity,  $C_p$  is the specific heat constant

pressure,  $\beta$  is the coefficient of thermal expansion,  $Q$  is the strength of the heat source .

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q = \frac{1}{L} \int_{-L_f}^{L_f} w dx \quad (12)$$

The boundary conditions are

$$u=0, w=0, T=T_1 \text{ on } x = -f \left( \frac{\delta z}{L} \right) \quad (13)$$

$$w=0, w=0, T=T_2 \text{ on } x = f \left( \frac{\delta z}{L} \right) \quad (14)$$

Invoking Rosseland approximation for radiation flux we get

$$q_r = -\frac{4\sigma^*}{\beta_R} \frac{\partial(T'^4)}{\partial x}$$

and linearising  $T'^4$  about  $T_e$  by using Taylor's expansion and neglecting higher order terms we get

$$T'^4 \approx 4T_e^3 T - 3T_e^4$$

where  $\sigma^*$  is the Stefan-Boltzman constant and  $\beta_R$  is the mean absorbing coefficient.

Eliminating the pressure from equations (8) & (9) and introducing the Stokes Stream function  $\psi$  as

$$u = -\frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \psi}{\partial x} \quad (15)$$

the equations (2.8)&(2.9),(2.10) in terms of  $\psi$  is

$$\frac{\partial \psi}{\partial z} \frac{\partial(\nabla^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial(\nabla^2 \psi)}{\partial z} = \mu \nabla^4 \psi + \beta g \frac{\partial(T - T_e)}{\partial x} - \left( \frac{\sigma \mu_e^2 H_0^2 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} \right) \nabla^2 \psi \quad (16)$$

$$\beta g \frac{\partial(T - T_e)}{\partial x} - \left( \frac{\sigma \mu_e^2 H_0^2 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} \right) \nabla^2 \psi$$

$$\rho C_p \left( \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} \right) = k_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q(T_2 - T) + \frac{16\sigma^* T_e^3}{3\beta_R} \frac{\partial^2 T}{\partial x^2} \quad (17)$$

$$Q(T_2 - T) + \frac{16\sigma^* T_e^3}{3\beta_R} \frac{\partial^2 T}{\partial x^2}$$

On introducing the following non-dimensional variables

$$(x', z') = (x, z) / L, \psi' = \frac{\psi}{qL}, \theta = \frac{T - T_2}{T_1 - T_2}$$

the equation of momentum and energy in the non-dimensional form are

$$\nabla^4 \psi - M_1^2 \nabla^2 \psi + \frac{G}{R} \left( \frac{\partial \theta}{\partial x} \right) = R \left( \frac{\partial \psi}{\partial z} \frac{\partial(\nabla^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial(\nabla^2 \psi)}{\partial z} \right) \quad (18)$$

$$R \left( \frac{\partial \psi}{\partial z} \frac{\partial(\nabla^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial(\nabla^2 \psi)}{\partial z} \right)$$

$$PR\left(\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}\right)=$$

$$\left(\left(1+\frac{4}{3N}\right)\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)-\alpha\theta \tag{19}$$

where  $G = \frac{\beta g \Delta T_e L^3}{\nu^2}$  (Grashof Number)

$$M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{\nu^2}$$
 (Hartman Number)

$$M_1^2 = \frac{M^2 \sin^2(\alpha)}{1+m^2} \quad R = \frac{qL}{\nu}$$
 (Reynolds Number)

$$P = \frac{\mu C_p}{K_f}$$
 (Prandtl Number)

$$\alpha = \frac{QL^2}{K_f}$$
 (Heat Source Parameter)  $N = \frac{\beta_R k_f}{4\sigma^* T_e^3}$

(Radiation parameter)

$$P_1 = \frac{3NP}{3N+4} \quad \alpha_1 = \frac{3N\alpha}{3N+4}$$

The corresponding boundary conditions are

$$\psi(f) - \psi(-f) = 1$$

$$\frac{\partial\psi}{\partial z} = 0, \frac{\partial\psi}{\partial x} = 0, \theta = 1 \quad \text{at } x = -f(\delta z)$$

$$\frac{\partial\psi}{\partial z} = 0, \frac{\partial\psi}{\partial x} = 0, \theta = 0 \quad \text{at } x = +f(\delta z)$$

### 3. Analysis of the flow

Introduce the transformation such that

$$\bar{z} = \delta z, \frac{\partial}{\partial z} = \delta \frac{\partial}{\partial \bar{z}}$$

Then  $\frac{\partial}{\partial z} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{z}} \approx O(1)$

For small values of  $\delta \ll 1$ , the flow develops slowly with axial gradient of order  $\delta$  and hence we take  $\frac{\partial}{\partial \bar{z}} \approx O(1)$ .

Using the above transformation the equations reduce to

$$F^4\psi - M_1^2 F^2\psi + \frac{G}{R}\left(\frac{\partial\theta}{\partial x}\right) =$$

$$\delta R\left(\frac{\partial\psi}{\partial \bar{z}}\frac{\partial(F^2\psi)}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial(F^2\psi)}{\partial \bar{z}}\right) \tag{20}$$

$$\delta PR\left(\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}\right)=$$

$$\left(\left(1+\frac{4}{3N}\right)\frac{\partial^2 T}{\partial x^2}+\delta^2\frac{\partial^2 T}{\partial z^2}\right)+\alpha \tag{21}$$

where

$$F^2 = \frac{\partial}{\partial x^2} + \delta^2 \frac{\partial}{\partial \bar{z}^2}$$

Assuming the slope  $\delta$  of the wavy boundary to be small we take

$$\psi(x, z) = \psi_0(x, z) + \delta\psi_1(x, z) + \delta^2\psi_2(x, z) + \dots$$

$$\theta(x, z) = \theta_0(x, z) + \delta\theta_1(x, z) + \delta^2\theta_2(x, z) + \dots \tag{22}$$

$$\eta = \frac{x}{f(\bar{z})}$$

Let  $\eta = \frac{x}{f(\bar{z})}$  (23)

Substituting (22) in equations (20) & (21) and using (23) and equating the like powers of  $\delta$  the equations and the respective boundary conditions to the zeroth order are

$$\frac{\partial^2\theta_0}{\partial\eta^2} - (\alpha_1 f^2)\theta_0 = 0 \tag{24}$$

$$\frac{\partial^4\psi_0}{\partial\eta^4} - (M_1^2 f^2)\frac{\partial^2\psi_0}{\partial\eta^2} = -\frac{Gf^3}{R}\left(\frac{\partial\theta_0}{\partial\eta}\right) \tag{25}$$

with

$$\psi_0(+1) - \psi_0(-1) = 1$$

$$\frac{\partial\psi_0}{\partial\eta} = 0, \frac{\partial\psi_0}{\partial\bar{z}} = 0, \theta_0 = 1 \quad \text{at } \eta = -1$$

$$\frac{\partial\psi_0}{\partial\eta} = 0, \frac{\partial\psi_0}{\partial\bar{z}} = 0, \theta_0 = 0 \quad \text{at } \eta = +1$$

(3.7)

and to the first order are

$$\frac{\partial^2\theta_1}{\partial\eta^2} = PRf\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial\theta_0}{\partial\bar{z}} - \frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial\theta_0}{\partial\eta}\right)$$

$$\frac{\partial^4\psi_1}{\partial\eta^4} - (M_1^2 f^2)\frac{\partial^2\psi_1}{\partial\eta^2} = -\frac{Gf^3}{R}\left(\frac{\partial\theta_1}{\partial\eta}\right) +$$

$$Rf\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial^3\psi_0}{\partial\bar{z}^3} - \frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial^3\psi_0}{\partial x\partial\bar{z}^2}\right) \tag{26}$$

with

$$\begin{aligned} \psi_1(+1) - \psi_1(-1) &= 0 \\ \frac{\partial \psi_1}{\partial \eta} = 0, \quad \frac{\partial \psi_1}{\partial \bar{z}} = 0, \quad \theta_1 = 0 \quad \text{at } \eta = -1 \\ \frac{\partial \psi_1}{\partial \eta} = 0, \quad \frac{\partial \psi_1}{\partial \bar{z}} = 0, \quad \theta_1 = 0 \quad \text{at } \eta = +1 \end{aligned} \tag{27}$$

#### 4. Shear stress and nusselt number

The shear stress on the channel walls is given by

$$\tau = \frac{(f^2(1+f'^2)\psi_{0,\eta\eta} + \delta(f^2(1+f'^2)\psi_{1,\eta\eta} - (2f'/f)\psi_{0,x\eta})) + O(\delta^2)}{(1+f'^2)}$$

and the corresponding expressions are

$$(\tau)_{\eta=+1} = \frac{(f^2(1+f'^2)b_{59} + \delta(f^2(1+f'^2)b_{65} - (2f'/f)b_{61})) + O(\delta^2)}{(1+f'^2)}$$

$$(\tau)_{\eta=-1} = \frac{(f^2(1+f'^2)b_{60} + \delta(f^2(1+f'^2)b_{66} - (2f'/f)b_{62})) + O(\delta^2)}{(1+f'^2)}$$

The rate of heat transfer (Nusselt Number) on the walls has been calculated using the formula

$$Nu = \frac{1}{f(\theta_m - \theta_w)} \left( \frac{\partial \theta}{\partial \eta} \right)_{\eta=\pm 1}$$

where

$$\theta_m = 0.5 \int_{-1}^1 \theta d\eta$$

$$(Nu)_{\eta=+1} = \frac{1}{f\theta_m} (a_{78} + \delta(a_{76} + a_{77}))$$

$$(Nu)_{\eta=-1} = \frac{1}{f(\theta_m - 1)} (a_{79} + \delta(a_{77} - a_{76}))$$

$$\theta_m = a_{80} + \delta a_{81}$$

#### 5. Results and discussion of the numerical results

We investigate the effect of Hall Currents and radiation effect on convective heat transfer flow of a viscous electrically conducting fluid in a vertical wavy channel in the presence of heat sources. The walls are maintained at constant temperatures. The velocity and temperature are analyzed for different values. The variation of 'w' with Hartmann 'M' and Hall parameter 'm' shows that higher the Lorentz force larger |w| in the flow region. An increase in m £ 2.5 accelerates 'w' and for further increase in m³ 3.5 we notice a depreciation in the axial velocity in flow region (Fig. 1). The depreciation for smaller values of N is remarkable and marginal for higher values of 'N' (Fig. 2). The variation of 'u' with reference to M and m shows that higher the strength of the magnetic field larger 'u' in the flow region. An increase in the Hall parameter m £ 2.5 enhances 'u' in the flow region and depreciates with higher m³ 3.5 (Fig. 3). Thus the presence of the radiative heat transfer depreciates the secondary velocity in the flow region (Fig. 4). The non-dimensional temperature distribution (q) is shown in Figs.5 and 6 for different parameter values. From Fig.5 it is find that higher the strength of the magnetic field (M £ 6) smaller the axial temperature and for further higher strength (M³ 10) larger the actual temperature. An increase in Hall parameter m £ 2.5 results in a depreciation in the axial temperature and for further values of m³ 3.5 we notice an enhancement in the axial temperature everywhere in the flow region. It is observed that higher the constriction of the channel walls larger the axial temperature in flow region. An increase in the radiation parameter N leads to depreciation in the axial temperature (Fig. 6).

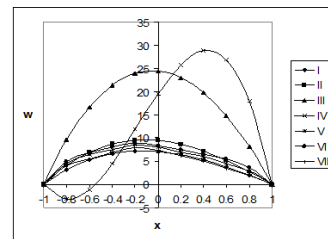


Fig. 1. Variation of w with M, m

	I	II	III	IV	V	VI	VII
M	2	4	6	10	0.2	0.2	0.2
m	0.5	0.5	0.5	0.5	1.5	2.5	3.5

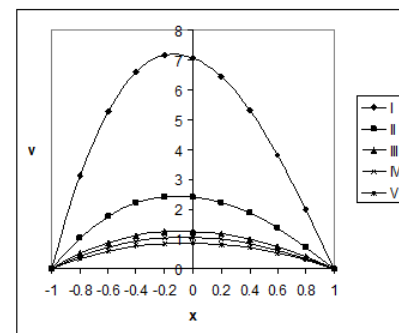


Fig. 2. w with N

	I	II	III	IV	V
N	0.5	1.5	5	10	100

Table 1  
 Shear stress ( $\tau$ ) at  $\eta = +1$

G	I	II	III	IV	V	VI	VII
$10^3$	0.588	1.177	12.723	0.259	0.689	0.594	0.602
$5 \times 10^3$	0.175	3.046	32.031	0.075	0.275	1.464	1.532
$-10^3$	-0.096	-1.316	-13.020	-0.066	-0.961	-0.612	-0.639
$-5 \times 10^3$	-0.212	-3.186	-32.328	-0.202	-0.262	-1.504	-1.569
M	2	4	6	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	4	6

Table 2  
 Shear stress ( $\tau$ ) at  $\eta = +1$

G	I	II	III	IV	V	VI
$10^3$	0.110	0.059	0.635	0.470	0.509	0.525
$5 \times 10^3$	0.321	0.175	1.605	1.203	1.299	1.492
$-10^3$	-0.170	-0.096	-0.659	-0.507	-0.546	-0.563
$-5 \times 10^3$	-0.380	-0.212	-1.630	-1.241	-1.337	-1.429
$\beta$	-0.3	-0.5	-0.7	-0.5	-0.5	-0.5
$N_I$	0.5	0.5	0.5	1.5	5	10

Table 3  
 Shear stress ( $\tau$ ) at  $\eta = -1$

G	I	II	III	IV	V	VI	VII
$10^3$	-16.63	-28.49	-38.49	-16.9	-17.21	-15.39	-14.36
$5 \times 10^3$	-15.50	-253.96	-39.33	-15.5	-15.50	-13.49	-12.30
$-10^3$	-17.39	-26.22	-40.84	-17.39	-18.39	-15.08	-13.10
$-5 \times 10^3$	-15.96	-25.75	-39.06	-14.96	-15.96	-14.08	-125.98
M	2	4	6	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	4	6

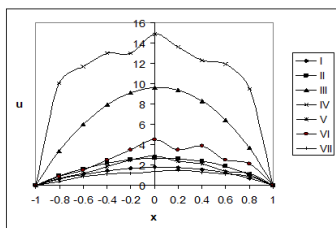


Fig. 3. Variation of  $u$  with  $M, m$

	I	II	III	IV	V	VI	VII
M	2	4	6	10	0.2	0.2	0.2
m	0.5	0.5	0.5	0.5	1.5	2.5	3.5

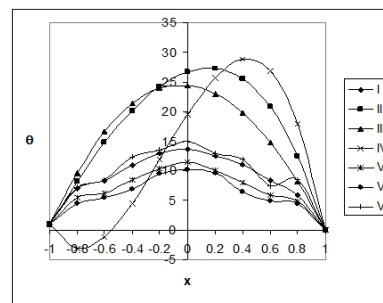


Fig. 5. Variation of  $\theta$  with  $M, m$

	I	II	III	IV	V	VI	VII
M	2	4	6	10	0.2	0.2	0.2
m	0.5	0.5	0.5	0.5	1.5	2.5	3.5

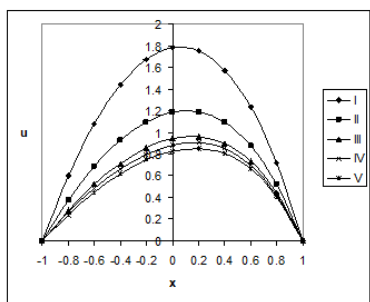


Fig. 4.  $u$  with  $N$

	I	II	III	IV	V
N	0.5	1.5	5	10	100

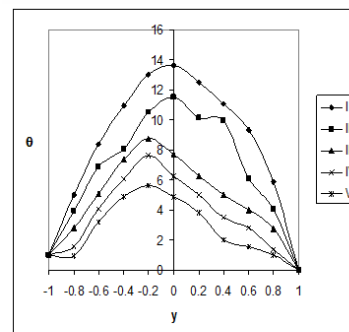


Fig. 6.  $\theta$  with  $N$

	I	II	III	IV	V
N	0.5	1.5	5	10	100

Table 4  
Shear stress ( $\tau$ ) at  $\eta = -1$

G	I	II	III	IV	V	VI
$10^3$	-17.46	-16.63	-19.21	-19.38	-15.17	-15.56
$5 \times 10^3$	-14.137	-15.50	-18.39	-14.69	-12.91	-12.02
$-10^3$	-15.57	-17.39	-19.76	-16.88	-15.06	-14.82
$-5 \times 10^3$	-14.37	-16.96	-18.53	-14.69	-12.82	-12.00
$\beta$	-0.3	-0.5	-0.7	-0.5	-0.5	-0.5
$N_1$	0.5	0.5	0.5	1.5	5	10

Table 5  
Nusselt number (Nu) at  $\eta = +1$

G	I	II	III	IV	V	VI	VII
$10^3$	0.779	2.690	4.717	0.679	0.579	-0.175	-0.295
$5 \times 10^3$	1.975	6.831	8.015	1.675	1.475	-0.410	-0.711
$-10^3$	-0.816	-2.829	-3.014	-0.616	-0.516	0.138	0.258
$-5 \times 10^3$	-2.013	-6.969	-4.312	-1.812	-1.613	0.373	0.674
M	2	4	6	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	4	6

Table 6  
Nusselt number (Nu) at  $\eta = +1$

G	I	II	III	IV	V	VI
$10^3$	0.201	0.779	1.60	-0.157	-0.298	-0.311
$5 \times 10^3$	0.546	1.975	4.02	-0.364	-0.716	-0.749
$-10^3$	-0.260	-0.816	-1.63	0.119	0.260	0.274
$-5 \times 10^3$	-0.606	-2.012	-4.05	0.327	0.679	0.712
$\beta$	-0.3	-0.5	-0.7	-0.5	-0.5	-0.5
$N_1$	0.5	0.5	0.5	1.5	5	10

Table 7  
Nusselt number (Nu) at  $\eta = -1$

G	I	II	III	IV	V	VI	VII
$10^3$	15.58	10.06	5.73	4.29	3.97	14.68	13.71
$5 \times 10^3$	16.71	14.88	6.56	5.71	4.71	15.89	14.30
$-10^3$	14.88	13.60	5.34	4.88	3.88	13.83	12.38
$-5 \times 10^3$	15.26	14.58	6.31	5.26	4.26	14.52	13.98
M	2	4	6	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	4	6

Table 8  
Nusselt number (Nu) at  $\eta = -1$

G	I	II	III	IV	V	VI
$10^3$	9.16	15.58	18.54	8.20	9.27	9.19
$5 \times 10^3$	13.87	15.71	18.33	13.49	12.11	12.36
$-10^3$	11.68	14.88	17.92	11.83	9.89	9.63
$-5 \times 10^3$	12.13	15.26	18.17	13.02	12.59	11.82
$\beta$	-0.3	-0.5	-0.7	-0.5	-0.5	-0.5
$N_1$	0.5	0.5	0.5	1.5	5	10

The stress ( $t$ ) at  $h = \pm 1$  are evaluated for different values of G, M, m, a, b,  $N_1$ , R, x and are shown in tables 1-8. The variation of  $t$  with Grashof 'G' shows that an increase in  $G > 0$  reduces  $|t|$  at  $h = \pm 1$  while it enhances at  $h = +1$  and depreciates at  $h = -1$  with increase in  $|G|$ . The variation of  $t$  with M shows that higher the strength of magnetic field larger the  $|t|$  at both the walls. An increase in the Hall parameter  $m \propto 1.5$  reduces  $|t|$  and enhances with higher  $m \propto 2.5$  at  $h = +1$  and at  $h = -1$   $|t|$  enhances with 'm' for all 'G'. With reference variation of  $t$  with 'a' reveals that  $|t|$  enhances at  $h = +1$  and reduces at  $h = -1$ . With increase in the strength of heat source (tables 1 and 3). The influence of surface geometry on the stress is shown in tables 2 and 4. Higher the constriction of the channel walls

smaller  $|t|$  and for further lowering of the constriction larger  $|t|$  at  $h = +1$  and at  $h = -1$  larger  $|t|$  for all G. An increase in the radiation parameter  $N_1$  results an enhancement of  $|t|$  at  $h = +1$  while at  $h = -1$ ,  $|t|$  enhances with  $N_1 \propto 1.5$  and depreciates with higher  $N_1 \propto 5$ .

The average Nusselt number (Nu) which measures the rate of heat transfer across the boundaries is shown in tables 5-8 for different variations. It is observed that the rate of heat transfer enhances with increase in  $|G|$  at  $h = \pm 1$ . An increase in M enhances  $|Nu|$  at  $h = +1$  and at  $h = -1$ ,  $|Nu|$  reduces with  $M \propto 4$  and enhances with  $M \propto 6$ . Also the rate of heat transfer depreciates with increase in Hall parameter 'm'. With reference of Nu with heat source parameter 'a', we find a decay in the

magnitude of Nu with increase in 'a', at both the walls (tables 5 and 7). The influence of surface geometry on Nu is exhibited in tables 6 and 7. Higher the constriction of the channel walls larger the rate of heat transfer at both the walls. An increase in the radiation parameter  $N_{1-1} \leq 1.5$  reduces |Nu| and enhances with higher  $N_{1-3} \leq 5$  and at  $h = -1$  the rate of heat transfer decays in magnitudes (tables 6 and 8).

## 6. Conclusion

This paper presented radiation effect on convective heat transfer in a vertical wavy channel with Hall effects.

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