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A Study on Geometry Measures of Hilbert Spaces

J. Geetha[1], K. Asha Devi[2]
[1]M. Phil. Scholar, Dept. of Mathematics, Ponnaiyah Ramajayam Inst. of Science and Tech., Thanjavur, India
[2]Assistant Professor, Dept. of Mathematics, Ponnaiyah Ramajayam Inst. of Science and Tech., Thanjavur, India

***Abstract*: In this paper, we investigate the geometry of Hilbert spaces and its related theorems are proved. This 3D space was applied on geophysical vibration signals from the process of drilling of rock.**

***Keywords*: Hilbert Spaces, inner product space, orthonormal.**

1. Introduction

The highest degree of generalization and abstraction of the physical space represent the classes of functional spaces called Hilbert spaces. The definition of Hilbert space is the following: Hilbert space is a complete space with inner product. It shows infinite-dimensional and complex. Hilbert Spaces are Inner Product Spaces, with rich geometric structures because of the orthogonality of vectors in the space. Hilbert space is a structure which combines the familiar ideas from vector spaces with the right ideas of analysis to form a useful context for handling mathematical problems of a wide variety [1].

2. Mathematical formulation

Definition: 1.1

An inner product on a complex linear space **L** is a function φ from **L** × **L** to C such that

1. $\phi(\propto_1 f_1 + \propto_2 f_2, g) = \propto_1 \phi(f_1, g) + \propto_2 \phi(f_2, g)$ for $\propto_1, \propto_2$ in C and $f_1, f_2, \ g$ in **L**
2. $\phi(f, \beta_1 g_1 + \beta_2 g_2) = \overline{\beta_1}\phi(f, g_1) + \overline{\beta_2}\phi(f, g_2)$ for β_1, β_2 in C and f, g_1, g_2 in **L**
3. $\phi(f, g) = \overline{\phi(g, f)}$ for f and g in **L** and
4. $\phi(f, f) \geq 0$ for f in **L** and $\phi(f, f) = 0$ if and only if $f = 0$

A linear space equipped with an inner. Product is said to be and inner product space.

Lemma: 1.2

If **L** is an inner product space with the inner product ϕ then

$$\phi(f, g) = \frac{1}{4}\phi(f+g, f+g) - \phi(f-g, f-g) + i\phi(f+ig, f+ig) - i\phi(f+ig, f+ig)$$

For f and g in **L**.

Example:

Let C^n denote the collection of collection of complex ordered n-tuples

$$\{x: x = (x_1, x_2, \dots \dots, x_n)\}$$

Inner product space in C^n

$$(x, y) = \sum_{i=1}^{n} x_i \overline{y_i}$$

and satisfies the eulidean norm

$$||x||_2 = (\sum_{i=1}^{n} |x_i|^2)^{1/2}$$

let $\{x^k\}_{k=1}^{\infty}$ is a cauchy sequence in C^n

$$|x_i^k - x_i^m| \leq ||x^k - x^m||^2$$

$$x = (x_1, x_2, \dots \dots \dots x_k) \quad where\ x_i = \lim_{k\to\infty} x_i^k$$

Thus x in C^n and

$\lim_{k\to\infty} x^k = x$ in the norm of C^n

$\Rightarrow C^n$ is Hilbert space

$l^2(z^+)$ is also Hilbert space

$l^2(z^+)$ collection of complex functions

ϕ on z^+ such that $\sum_{n=0}^{\infty} |\phi(n)|^2 < \infty$

Definition: 1.3

If L is an inner product space, then the norm $\|\ \|$ on L associated with the inner product is defined by $\|f\| = (f, f)^{1/2}$ for f in L.

Cauchy – Schwarz inequality: 1.4

If f and g are in the inner product space L then $|(f, g)| \leq \|f\|\|g\|$

Proposition: 1.5

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1. If L is an inner product space then || || defines a norm on L
2. In an inner product space, the inner product is continuous.

Definition: 1.6

In the inner product space L two vectors f and g are said to be orthogonal denoted $f \perp g$ if $(f, g) = o$

A subset **S** of L is said to be orthogonal if $f \perp g$ for f and g in S orthogonal if, in addition $\|f\| = 1$ for f in **S**.

Pythagorean Theorem: 1.7

If $\{f_1, f_2, \ldots \ldots . f_n\}$ is an orthogonal subset of the inner product space L, then

$$||\sum_{i=1}^{n} f_i||^2 = \sum_{i=1}^{n} ||f_i||^2$$

Parallelogram law: 1.8

If f and g are in the inner product space L, then

$$||f + g||^2 + ||f - g||^2 = 2\,||f||^2 + 2\,||g||^2$$

Definition: 1.9

A Hilbert space is a complex linear space which is complete in the metric induced by the norm.

The space H^2: 1.10

Let **T** denote the unit circle, U the normalized lebesgue measure **T** and $L^2(T)$ the Hilbert space defined with respect to μ. The corresponding hardy space H^2 is defined as the closed subspace.

$$\{f \in L^2(T);\ \frac{1}{2\pi}\chi \int_0^{2\pi} f\chi_n dt = 0 \quad for \quad n = 1,2,3, \ldots \ldots\}$$

where χ_n is the function $\chi_n(e^{it}) = e^{int}$

a slight variation of this definition is

$$\{f \in L^2(\boldsymbol{T});\ (f, \chi_n) = 0 \quad for \quad n = -1, -2, -3, \ldots \ldots .\}$$

Definition: 1.11

If M is a subset of the Hilbert space H; then the orthogonal complement of M denoted M', is the set of vectors in H orthogonal to every vector in H.

Riesz Representation theorem: 1.12

If ϕ is a bounded linear functional on H;

then there exist a unique of in H such that

$\phi(f) = (f, g) \quad for \;\; g \;\&\; f \; in$ H.

Definition: 1.13

A subset $\{e_\propto\}_{\propto \in A}$ of the Hilbert space H is said to be an normal basis if it is orthonormal and the smallest closed subspace containing it is H.

Theorem: 1.14

Every Hilbert space $\{\neq \{0\}\}$ possesses an orthonormal basis.

Definition: 1.15

If H is a Hilbert space then the dimension of H denoted dim it, is the cardinality of any orthonormal basis for H.

Theorem: 1.16

Two Hilbert spaces are isomorphic if and only if their dimension are equal.

Example: 1.17

1. For C^n with n-tuples $\{(1, 0, 0 \ldots \ldots 0), (0,1,00, \ldots \ldots 0) \ldots \ldots . (0,00 \ldots \ldots .1)\}$ form an orthonormal basis dim $C^n = n$
2. Let $f \in l^2(z^+)$ defined by
$$f = \sum_{n=0}^{\infty} f(n) e_n$$
$\Rightarrow \{e_n\}_{n=0}^{\infty}$ is an orthonormal basis for $l^2(z^+)$
3. Hilbert space
R^n is an inner product space with
Inner product $(x, y) = \sum_{k=1}^{n} x_k y_k$

Orthonormal basis 1.18

Let $H = l^2$ and for n =1,2, ……

$$u_n = (0, \ldots . .0,1,0,0, \ldots \ldots)$$

Where 1 occurs only in the n^{th} entry.

Then $\{u, u_2, \ldots . .\}$ is an orthonormal set in H. If $x \in H$

$x(n) =< x, u_n >= 0$ for all n_1 then $x = 0$

Hence $\{u_n: n = 1,2, \ldots$ …. is an orthonormal basis for H.

3. Conclusion

The study of Hilbert spaces and its related theorems are discussed. Hilbert spaces provide special geometrical properties are discussed and it is useful for drilling of rocks.

References

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