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Efficiency of Modified Product Estimators Under a Super Population Model

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***Abstract*: In this paper efficiency of transformed product type estimators suggested by Pandey and Dubey (1988), Upadhyay and Singh (1999), Singh (2003, 2004) have been studied under a super population model suggested by Durbin (1959). It is found that Singh (2003) estimator performs better among all these estimators.**

***Keywords*: Auxiliary information, simple random sampling without replacement (SRSWOR), finite population correction, Variance, order of approximation, Bias, Mean square error (MSE), Relative efficiency.**

1. Introduction

Product method of estimation is well known technique for estimating the population mean of a study character when population mean of an auxiliary character is known and it is negatively correlated with study character. Similarly, ratio method of estimation is used when study variable is highly correlated with auxiliary variable.

Mohanty and Das (1971) introduced the use of transformation as a tool for the reduction of Mean square error and bias of the ratio estimator of the population mean simultaneously. It has been shown that replacing X by (X + α/β) the bias of ratio estimator becomes zero when Y = α + β X is the regression line of y on x in the population. Moreover, a study on the use of transformation on the product estimator reveals that an increment to each value of the auxiliary variable by an amount reduces the value of the mean square error of the product estimator of the population mean to a minimum. On the other hand, the bias of the above mentioned product estimator can be reduced by changing each value of the auxiliary variable by X by a sufficient large amount. Related works in this area are available in the papers of Kulkarni (1978), Sisodia and Dwivedi (1981).

Let $U=(u_1,u_2\ldots\ldots\ldots u_N)$ be the finite population of N units, y and x be the character under study and auxiliary character, respectively. It is assumed that y and x are negatively correlated. Let $y_k>0$, and $x_k>0$ be the values of y and x for the i-th (i=1, 2………..N) unit in the population. From the population U, a simple random sample of size n is drawn without replacement. Let and be the population means and sample means of y and x respectively. The usual product estimator for the population mean is defined as

$$\bar{y}_p = \frac{\bar{y}.\bar{x}}{\bar{X}} \qquad (1.1)$$

It is well known that product estimator $\bar{y}_p$ will estimate $\bar{Y}$ in large samples more precisely than sample mean $\bar{y}$ if

$$\rho \le -\frac{c_x}{2c_y} \qquad (1.2)$$

Where ρ is correlation coefficient between y and x and c_x and c_y are coefficients of variations of y and x respectively.

To find more precise estimates Searls (1964) used coefficient of variation (CV) of study character at estimation stage. In practice this CV is seldom known. Motivated by Searls (1964) work, Sisodia and Dwivedi (1981) used the known CV of the auxiliary character for estimating population mean of a study character in ratio method of estimation. Following the work of Sisodia and Dwivedi (1981), Pandey and Dubey (1988) proposed a modified product estimator for population mean using known CV of an auxiliary character. Recently, Upadhyaya and Singh (1999) proposed new product estimators using known CV and coefficient of Kurtosis (CK) of an auxiliary character. All these authors have used known CV and CK of an auxiliary character in additive form to sample and population means of the same character. It could be noticed that CV and CK are unit free constants therefore their additions may not be justified. Further if coefficient of variation and population mean of an auxiliary character are known, standard deviation of the same auxiliary character is automatically known and use of standard deviation in additive form is more justified. Singh (2003) proposed new product estimators using known standard deviation (SD) of an auxiliary character, coefficient of Skewness (CS) and coefficient of Kurtosis (CK) of an auxiliary character.

Pandey and Dubey (1988) estimator

$$t_2 = \bar{y}\left[\frac{\bar{x}+C_x}{\bar{X}+C_x}\right] \qquad (1.3)$$

Upadhyaya and Singh (1999) estimator

$$t_3 = \bar{y}\left[\frac{\beta_2(x)\bar{x} + C_x}{\beta_2(x)\bar{X} + C_x}\right] \quad (1.4)$$

$$t_4 = \bar{y}\left[\frac{C_x\bar{x} + \beta_2(x)}{C_x\bar{X} + \beta_2(x)}\right] \quad (1.5)$$

Singh (2003) estimator

$$t_5 = \bar{y}\left[\frac{\bar{x} + \sigma_x}{\bar{X} + \sigma_x}\right] \quad (1.6)$$

$$t_6 = \bar{y}\left[\frac{\beta_1(x)\bar{x} + \sigma_x}{\beta_1(x)\bar{X} + \sigma_x}\right] \quad (1.7)$$

$$t_7 = \bar{y}\left[\frac{\beta_2(x)\bar{x} + \sigma_x}{\beta_2(x)\bar{X} + \sigma_x}\right] \quad (1.8)$$

Singh et al. (2004) estimator

$$t_8 = \frac{\bar{x} + \beta_2(x)}{\bar{X} + \beta_2(x)} \quad (1.9)$$

Next, we study properties of above transformed estimators under Durbin (1959) Model, where the relation between y and x is of the form

$$y_i = \alpha + \beta x_i + e_i ;\ \beta < 0 \quad (1.10)$$

$$E[e_i / x_i] = 0$$

$$E[e_i e_j / x_i x_j] = 0 \qquad \text{for } i \neq j$$

$$V[e_i / x_i] = n\delta \quad [\delta \text{ is a constant of order } n^{-1}]$$

And the variate x_i / n have the gamma distribution with the parameter m= nh.

2. Bias and mean square errors

All the above estimators may be seen to be particular case of transformed product estimator

$$\bar{y}_{Tp} = \frac{\bar{y}(\bar{x} + A)}{\bar{X} + A} \quad (2.1)$$

where A is constant. This estimator has been discussed by Dubey (1988). Using Durbin (1959) model, the bias of $\bar{y}_p$ and $\bar{y}_{TP}$

$$B(\bar{y}_p) = \beta \quad (2.2)$$

$$B(\bar{y}_{Tp}) = \frac{\beta m}{m + A} \quad (2.3)$$

respectively. Comparing (2.2) and (2.3), we find that for $A > 0$

$$|B(\bar{y}_{Tp})| < |B(\bar{y}_p)| \quad (2.4)$$

The MSE's of $\bar{y}_p$ and $\bar{y}_{Tp}$, under model are respectively given by

$$MSE(\bar{y}_p) = \frac{1}{m}\left[\alpha^2 + \beta^2(4m^2 + 11m + 6) + 4\alpha\beta(m+1) + \delta(m+1)\right] \quad (2.5)$$

$$MSE(\bar{y}_{Tp}) = \alpha^2\left(\frac{m}{(m+A)^2}\right) + \beta^2 m\left(1 + \frac{(m+1)(5m+4A+6)}{(m+A)^2} - \frac{2m}{(m+A)}\right) + 2\alpha\beta m\left(\frac{3m+2A+2}{(m+A)^2} - \frac{1}{(m+A)}\right) + \delta\left(1 + \frac{m}{(m+A)^2}\right) \quad (2.6)$$

The variance of $\bar{y}$ under above model is obtained as,

$$V(\bar{y}) = \beta^2 m + \delta \quad (2.7)$$

It can be seen that expressions of Bias and MSE of usual product estimator may easily be obtained from (2.3) and (2.6) by taking A=0. Considering $A = C_x$, $\{C_x / \beta_2(x)\}$, $\{\beta_2(x)/C_x\}$, σ_x, $\{\sigma_x / \beta_1(x)\}$, $\{\sigma_x / \beta_2(x)\}$, $\beta_2(x)$ in (2.1), the estimators $t_2, t_3, t_4, t_5, t_6, t_7, t_8$ may easily be obtained. Similarly, the expression of their bias and MSE of these estimators are obtained by substituting corresponding values of A in (2.3) and (2.4).

Theoretically, it is difficult to compare (2.5), (2.6) and (2.7) explicitly. Therefore we compare the efficiency of above estimators for various values of m, ρ and k.

3. Numerical illustration

We note that in terms of the model

$$\alpha = \bar{Y}[(k - \rho)/k]$$

$$\beta = \bar{Y}[\rho/km]$$

$$\delta = \bar{Y}^2[(1-\rho^2)/k^2 m]$$

$$k = C_x / C_y$$

The exact efficiencies of t_i (i=1, 2, …) relative to $\bar{y}$ are given by

$$E_i = \frac{V(\bar{y})}{MSE(t_i)} \times 100$$

Substituting the values of α, β and δ given efficiencies E_i can be expressed explicitly as a function of $k = C_x/C_y$, m = nh, ρ

Since the expressions for the relative efficiencies are complex, we evaluate these quantities for fixed h=1 and selected values of n, ρ, k and presented in the table:

Table 1
m=4.0, ρ= -0.50

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	71.11	82.05	82.05	36.78
E_2	32.07	108.27	270.98	75.23
E_3	30.86	103.23	230.53	58.92
E_4	38.85	128.94	421.67	1718.87
E_5	35.12	120	394.52	1172.19
E_6	35.12	120	394.52	127.19
E_7	31.92	107.61	265.31	72.72
E_8	37.41	126.85	457.7	1810.0

Table 2
m=4.0, ρ= -0.70

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	52.60	73.05	93.56	50.79
E_2	45.30	102.94	231.71	105.70
E_3	42.95	96.66	203.72	80.27
E_4	60.98	136.172	279.98	1061.40
E_5	51.69	119.11	299.47	271.28
E_6	51.69	119.11	299.47	271.28
E_7	44.99	102.11	227.94	101.72
E_8	57.10	115.0	313.57	847.80

Table 3
m=4.0, ρ= -0.80

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	52.28	66.66	95.23	60.60
E_2	51.69	90.63	181.03	119.14
E_3	48.40	84.44	162.08	89.86
E_4	76.94	128.63	215.10	641.09
E_5	61.16	107.44	224.77	303.95
E_6	61.16	107.44	224.77	303.95
E_7	51.25	89.80	178.52	114.55
E_8	69.97	120.81	234.65	741.96

Table 4
m=4.0, ρ=-0.90

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	46.37	59.92	93.56	73.05
E_2	52.78	74.89	135.16	126.42
E_3	48.83	69.42	122.66	95.82
E_4	87.75	112.79	163.83	402.56
E_5	64.79	90.39	164.07	298.25
E_6	67.79	90.39	164.07	298.25
E_7	52.24	74.15	133.51	121.68
E_8	77.06	103.81	173.13	530.68

Table 5
m=8.0, ρ=-0.50

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	92.08	104.06	92.08	35.26
E_2	67.03	234.47	349.96	55.82
E_3	66.52	231.72	321.34	51.28
E_4	73.05	237.36	943.07	766.30
E_5	70.25	246.55	657.93	116.56
E_6	71.92	245.85	923.04	243.18
E_7	67.74	237.99	396.45	63.42
E_8	70.95	247.22	765.79	149.52

Table 6
m=8.0, ρ= -0.70

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	85.92	109.47	126.83	49.05
E_2	86.42	208.91	440.67	77.55
E_3	85.47	206.67	409.41	70.33
E_4	98.53	206.71	528.30	1912.65
E_5	92.60	217.82	659.75	186.00
E_6	96.03	215.68	670.31	469.50
E_7	87.75	211.74	487.68	89.94
E_8	94.00	217.84	685.87	253.01

Table 7
m=8.0, ρ= -0.80

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	81.46	109.28	150.37	60.06
E_2	91.48	174.35	388.10	91.20
E_3	90.23	172.35	367.82	82.16
E_4	108.77	177.11	366.82	1890.80
E_5	99.89	183.27	492.89	234.21
E_6	104.87	172.98	463.13	632.23
E_7	93.25	176.92	463.13	106.89
E_8	101.89	183.82	417.06	326.91

Table 8
m=8.0, ρ=-0.90

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	76.42	107.13	179.07	76.42
E_2	89.97	137.50	301.45	106.06
E_3	85.57	135.80	290.30	95.00
E_4	108.05	144.83	258.60	1239.04
E_5	96.72	146.09	342.88	285.68
E_6	102.87	147.55	315.90	748.34
E_7	88.97	139.75	315.89	125.40
E_8	99.14	147.13	337.89	401.74

Table 9
m=16.0, ρ=-0.50

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	106.00	117.70	96.42	34.33
E_2	134.51	496.83	379.33	49.06
E_3	134.42	495.35	365.82	47.65
E_4	134.07	441.32	1990.52	306.04
E_5	135.36	500.00	767.38	87.49
E_6	133.59	427.11	2045.44	415.17
E_7	134.91	502.11	457.87	57.08
E_8	135.33	502.11	689.01	79.84

Table 10
m=16.0, ρ= -0.70

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	107.931	138.84	146.46	47.44
E_2	151.45	403.07	669.30	66.81
E_3	151.34	403.83	643.19	64.59
E_4	149.83	311.32	1152.50	684.67
E_5	152.28	376.25	1264.91	132.55
E_6	149.04	299.62	1001.50	1067.03
E_7	151.92	397.88	816.55	79.68
E_8	152.30	381.39	1175.33	118.65

Table 11
m=16.0, ρ= -0.80

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	107.52	149.81	193.23	58.22
E_2	146.16	312.00	728.88	78.74
E_3	146.01	312.75	706.01	75.94
E_4	145.94	244.07	702.84	1087.34
E_5	147.94	290.61	1054.58	166.52
E_6	145.24	235.76	617.37	1791.90
E_7	146.83	307.37	845.08	95.29
E_8	147.59	294.44	1031.85	147.32

Table 12
m=16.0, ρ= -0.90

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	106.21	160.58	277.59	74.89
E_2	128.81	231.61	639.18	93.22
E_3	128.62	232.11	627.27	89.65
E_4	130.78	188.11	446.23	16021.01
E_5	131.07	217.99	715.83	210.99
E_6	130.34	182.64	399.98	2378.81
E_7	129.67	228.61	689.91	114.58
E_8	130.88	220.41	723.91	184.37

Table 13
m=32.0, ρ=-0.50

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	114.09	125.25	98.31	33.84
E_2	254.20	1027.81	389.78	46.84
E_3	254.34	1026.97	384.42	45.43
E_4	235.68	825.94	2386.76	169.30
E_5	247.80	1004.89	715.68	71.54
E_6	220.03	594.06	3172.07	925.47
E_7	252.32	1032.15	466.97	52.90
E_8	250.65	1027.21	546.51	59.19

Table 14
m=32.0, ρ=-0.70

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	122.45	158.16	156.61	46.49
E_2	235.09	711.54	849.19	62.50
E_3	235.30	713.99	833.80	61.75
E_4	211.14	457.19	2838.71	307.61
E_5	226.14	607.40	1823.01	103.93
E_6	193.61	331.26	920.99	4493.16
E_7	232.34	679.36	1076.54	72.74
E_8	230.00	651.86	1317.60	82.97

Table 15
m=32.0, ρ=-0.80

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	126.18	180.18	219.78	56.98
E_2	204.62	496.23	1166.65	73.55
E_3	204.80	498.03	1146.47	72.63
E_4	184.37	329.17	1511.30	442.30
E_5	197.06	425.19	2065.07	128.23
E_6	169.49	247.95	556.11	863.84
E_7	202.29	473.37	1445.87	86.70
E_8	200.32	454.60	1697.80	100.08

Table 16
m=32.0, ρ=-0.90

	k=0.25	k=0.50	k=1.0	k=2.0
E_1	129.49	207.62	363.14	73.37
E_2	167.90	342.79	1260.71	87.39
E_3	168.03	343.91	1247.05	86.21
E_4	153.30	240.40	828.54	667.92
E_5	162.55	299.37	1498.34	160.92
E_6	142.16	188.40	365.49	4427.79
E_7	166.27	328.71	1412.81	104.54
E_8	164.87	317.25	1492.29	122.36

4. Conclusion

For all values of ρ, k no single estimator stands out. However for $\rho \leq -0.7$ and for small m, Upadhyaya and Singh estimator performs better, followed by Singh et al (2004) and for different value of ρ they perform better than usual product estimator.

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