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Innovative Techniques and Concepts for Teaching and Learning of Linear Programming Problem

R. Prabakaran[1], S. Jayaram[2], R. S. Abhiraami[3]
[1]*Assistant Professor, Department of Mathematics, Coimbatore Institute of Technology, Coimbatore, India*
[2,3]*PG Student, Department of Mathematics, Coimbatore Institute of Technology, Coimbatore, India*

***Abstract*: Linear programming is a mathematical modelling technique useful for guiding quantitative decisions in business planning, industrial engineering and to a lesser extent, in the social and physical sciences. It is the maximization or minimization of a specific performance index, usually of an economic nature like profit, subject to a set of linear constraints. The goal of this paper to utilize the concept of Simplex algorithm and an aspect of linear programming to allocate raw materials to competing variables (big loaf, giant loaf and small loaf) in bakery for the purpose of profit maximization. The analysis was carried out and the result showed that 1000 units of small loaf, 500 units of big loaf and 0 unit of giant loaf should be produced respectively in order to make a profit of N30000. From the analysis, it was observed that small loaf, followed by big loaf contribute objectively to the profit. Hence, more of small loafs and big loafs are needed to be produced and sold in order to maximize the profit.**

***Keywords*: Linear programming model, Simplex method, Decision variables, Optimal result.**

1. Introduction

Linear programming also called a linear optimisation is a method to achieve the best outcome (such as maximum profit or lowest cost) in mathematical model which are represented in the form of a linear relationship. Linear programming comes under the class of mathematical programming. A linear programming algorithm finds importance in the Polytope. The expression in a linear programming that is to be maximized or minimized is known as the objective functions. More precisely a linear programming forms a means of linearizing the objective functions, subject to linear equality constraints and linear inequality constraints. Linear programming in today's world finds applications in various fields. Most importantly used in mathematics field. It finds a lesser application in business economics and also finds relevance in engineering domain also. Industries also make use of linear programming models including transportation, energy, telecommunications, and manufacturing. It also had proven to be a supporting means in modelling diverse types of problems viz; planning, routing scheduling, assignment, and design.

A. History

During 1946–1947, a general linear programming formulation was developed which extended its application to use it for planning problems in the US Air Force. In 1947, the simplex method was invented by a mathematician, which efficiently tackled the linear programming problem for the first time in various aspects. Later the scientist went for a meeting and they discussed the problem with simplex method simplex method, and hence to overcome its limitations an improved methodologies called the DUALITY THEORY and the GAME THEORY came into existence. However, when using a simplex algorithm or duality or game theory as mentioned above, it takes just a minute to obtain the optimum solution by posing it as problem. The idea behind introducing a linear programming will consequently minimizes required number of steps and solutions to provide exclusivity. The linear programming problem initially were subjected to be solved in a polynomial time environment. During the passage of time a drastic breakthrough in the practical applications in this field was observed in 1984.As an improvement the existing practical linear programming problems were solved using Interior Point Method.

B. Uses

Linear programming finds numerous applications in several areas of mathematical modelling to achieve an optimised solution. In fact, some practical issues related to the operational research can be sorted out by expressing those problems as linear programming problems. Some special cases of linear programming, involving commodity flow problems multi commodity problems and the network flow issues are all handled using specialized algorithms. Numerous algorithms that needs to be optimised, works by considering LPP as a Sub solutions and solving on them. LPP are currently involved in the formation of microeconomics and are utilized in company management that includes aspects such as planning, production, transportation, technology and other issues. According to modern management theory, companies would like to maximize their profits focuses on minimizing the costs with a limited amount of resources. As a result, these above mentioned issues and their solutions can be formulated by characterising them as linear programming problem.

C. Integral Linear Program

A linear program in real variables is said to be INTEGRAL, if it has at least one optimal solution which is integral. Integral linear programs are important concept in the field of combinational optimization, as they provide some alternate characterization for a particular solution.

2. Representation of a Linear Programming Problem

Minimize $C_1 X_1 + C_2X_2 + C_3X_3 + \ldots\ldots\ldots C_nX_n = Z$

Subject to $a_{11}x_1 + a_{12}x_1 + \cdots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots\ldots a_{2n} x_n = b2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$

Where $x_1,\ x_2, \ldots\ x_n \geq 0$

In linear programming z, the expression being optimized, is called the objective function. The variables $x_1,\ x_2 \ldots x_n$ are called decision variables, and their values are subject to m + 1 constraints. A set of x1, x2 ...xn satisfying all the constraints is called a feasible point and the set of all such points is called the feasible region. The solution of the linear program must be a point $(x_1, x_2\ \ldots x_n)$ in the feasible region, or else not all the constraints would be satisfied. The following example is a useful tool in interpreting the feasible region for solving linear programming problems with two decision variables. The linear program is:

Minimize Z= $4x_1 + x_2 + x_3$= Subject to $3x_1 + x_2 \geq 10$ $x_1 + x_2 \geq 5$ $x_1 \geq 3x_1$, $x_2 \geq 0$.

We can plot these constraints in a graph. Since all these constraints are greater than or equal to, the shaded region above all three lines is the feasible region. The solution to this linear program lie in the shaded region. Therefore, z = 14 is the smallest possible value of z.

Some Basic Terms and Definitions

Solution: The set of values $x_1, x_2 \ldots x_n$ which satisfies the constraints of an LPP is defined as Solution.

Feasible Solution: Any solution to LPP, which satisfy the Non-negativity restriction of LPP is called feasible solution.

Optimal Solution: Any feasible solution which optimise the objective function of the LPP is called its Optimal Solution.

Slack Variable: If the constraints of the General LPP is represented as "less than or equal to", i.e. an Inequality, then we must introduce a non-negative variable to convert this inequality to an equality. These variables added to the constraints are called the Slack Variables. The physical meaning of this variable conveys the amount of unused resources.

Surplus Variable: If the constraints of the General LPP is an Inequality represented as "greater than or equal to", then we must introduce a non-negative variable to convert this inequality. Such variables are called Surplus Variables. The physical meaning of this variable conveys the amount over the required level.

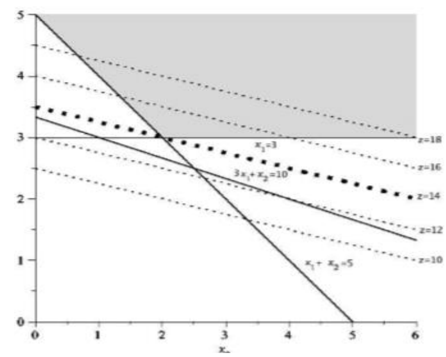

Fig. 1. Graphical representation

3. Standard form of a Linear Programming Model

The use of the simplex method to solve a linear programming problem requires that the problem be converted into its standard form. The standard form of a linear programming problem has the following properties. All the constraints should be expressed as equations by adding slack or surplus variables.

The right-hand side of each constraint should be made of non-negative (if not). This is done by multiplying both sides of the resulting constraints by -1.

The objective function should be of a maximization type.

Assumptions in a Linear Programming Problem

Before we start with the linear programming problems it is very important to have some prerequisites about various theorems and assumptions. For instance, there are 4 different theorems. These are:

1. *Proportionality:* The contribution of any variable to the objective function or constraints is proportional to that variable. This implies no discounts or economies to scale. For example, the value of 8x1 is twice the value of 4x1, no more or less.
2. *Additivity:* The contribution of any variable to the objective function or constraints is independent of the values of the other variables.
3. *Divisibility:* Decision variables can be fractions. However, by using a special technique called integer programming, we can bypass this condition. Unfortunately, integer programming is beyond the scope of this paper.
4. *Certainty: This* assumption is also called the deterministic assumption. This means that all parameters (all coefficients in the objective function and the constraints) are known with certainty. Realistically, however, coefficients and parameters are

often the result of guess-work and approximation. The effect of changing these numbers can be determined with sensitivity analysis.

4. Tools for solving linear programs

There are some Important Precursors to the Simplex Method. Linear programming is introduced to obtain the unique solutions to some linear programs. Theoretically when we say, finding a solution to a linear program is more important than the theory behind it. The most popular method of solving linear programs is called the Simplex algorithm, Big-M method, and two phase method.

The Simplex Method: The Simplex algorithm method is introduced to overcome the shortcomings of a method called "brute force" approach. Rather than checking all extreme points in a given region, the Simplex algorithm helps to selects arbitrary point at which we have to start. Each iteration in the given algorithm takes the system to the adjacent extreme point having a unique objective function value. The iterations will be repeated until there is no adjacent extreme points having a better objective function value. Hence we can conclude that system is at optimal. The most suitable way to implement a Simplex algorithm in practice is through formulating the quantities in a tabular arrangement. The best way to achieve this is by creating a matrix with a column for each variable, starting with the objective function value z.

Problems Associated with Simplex Method: Simplex algorithm will lead us to some special cases and exceptions. Consider a case where the initial table contains no starting basis. This problem occurs when a given constraint is either greater than or equal to form. In the case of an equality constraint, one cannot add a slack or an excess variable and therefore there will be no initial basic variable in that row. In the case of a "greater than or equal to" constraint, one must add an excess variable, which gives a −1 in that row instead of a +1.We have to deal with such problems. The methods that we adopt to solve such problems are Two-Phase Method and the Big-M Method. Both of these methods involve adding artificial variables that start out as basic variables but must eventually equal zero for original problem to be feasible.

The Big-M Method: First way to deal the problem aforementioned is called the Big-M Method. The following example will help illustrate this method:

Consider the problem

Minimize Z= $x_1 - 2x_2$ Subject to $x_1 + x_2 \geq 2$ $-x_1 + x_2 \geq 1$ $x_2 \leq 3x_1$,$x_2 \geq 0$.

The most positive coefficient in Row 0 is 2 + 2M. Thus we select that particular column for a pivot element and the ratio test is performed. Row 2 wins the ratio test. The term 1 + 2M is the most positive so that column is the pivot column. The following part of the Big-M Method is just similar to Simplex Method, except that a variable "M" floating around. This process continues for a few more steps until all the Row elements of the obtained result is positive, thus we can conclude that the feasible solution given is an optimal solution.

Consider the following problem:

Maximize Z= $2x_1 - x_2 + x_3$. Subject to $2x_1 + x_2 - 2x_3 \leq 8$; $4x_1 - x_2 + 2x_3 = 2$; $2x_1 + 3x_2 - x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$.

When we represent the above equation in a standard form all the given constraints will be equalities that is with only "equal to" (=) symbol. Adding a slack variable, s1, to the first constraint and subtracting an excess variable, e3, from the third constraint gives the initial Simplex table. Since there is no initial basis in the given constraints we can add two or more special variable called "ARTIFICIAL VARIABLE". a1 and a3 to the constraints. To correct this, add two more variables, called "artificial variables," a1 and a3. We must also keep in mind, adding two artificial variables and should to alter the initial objective function.

Two Phase Method:

In a Two-Phase Method, initially, the objective function has to be replaced with objective function from the original problem and we must ensure that all entries of Row is equal to zero. Secondly we must, delete all columns associated with artificial variables. The as a third step we should proceed as usual with the Simplex algorithm till we achieve optimal solutions. Now the prior concern is given to maximise the objective function. Thus a Two-Phase Method can be completed as follows;

- STEP 1: When there is no basis in original table, we should add artificial variables to those rows where there are no slack variables.
- STEP2: Formulate an objective function to minimize the slack variables.
- STEP3: Modify the table so that the Row 0 coefficients of the artificial variables are zero.

Case study on LPP:

Optimal Use of Raw Materials in Bakery

The linear programming is ranked top among the most important creative and scientific advancements for the applications in industrial raw material stocks. In the mid of 20^{th} century many companies adopted LPP methodology for their marketing. The major reason is that, many production companies face a lack of knowledge regarding proper utilisation of their available raw materials so as to earn maximum profit. So the use of linear programming would bring an appropriate quantitative decision-making at times. This decision production were based on the relationship between total raw materials used as input and its output. In order to avoid a biased decision that can lead to a reduced profit in future, such as price fluctuation and shortage of raw materials or available resources, the of linear programming model serve as a most powerful tools for several decision makers to achieve an effective decision. This application deals with the use of linear programming for optimal production in bakery, for the optimal production of its raw materials. The resulting model was proposed using a simplex algorithm. At the end of their analysis a conclusion was made that out of the nine product they manufacture using available raw materials only two product contribute most to their profit maximization. The bakery industry analysed this situation using the LPP as follows;

Assumptions

It is assumed that the raw materials required for production of bread are limited (scarce). It is assumed that an effective allocation of raw materials to the variables (big loaf, giant loaf and small loaf) will aid optimal production and at the same time maximizing the profit of then bakery. It is assumed that the qualities of raw materials used in bread production are standard (not inferior).

Data Analysis and Calculation

The data required for this research was collected and formulated. The data consist of total amount of raw materials (sugar, flour, yeast, salt, and wheat gluten and soybean oil) available for daily production of three different sizes of bread (big loaf, giant loaf and small loaf) and profit contribution per each unit size of bread produced. The content of each raw material per each unit product of bread produced are listed below.

Flour

Total amount of flour available = 300kg
Each unit of big loaf require 0.5kg of flour
Each unit of giant loaf requires 1kg of flour
Each unit of small loaf requires 0.34kg of flour

Sugar

Total amount of sugar available = 50kg
Each unit of big loaf requires 0.40g of sugar
Each unit of giant loaf requires 0.50g of sugar
Each unit of small loaf requires 0.30g of sugar

Yeast

Total amount of yeast available = 20kg
Each unit of big loaf requires 0.2kg of yeast
Each unit of giant loaf requires 0.2kg of yeast
Each unit of small loaf requires 0.2kg of yeast

Salt

Total amount of salt available = 1kg
Each unit of big loaf requires 0.002g of salt
Each unit of giant loaf requires 0.02g of salt
Each unit of small loaf requires 0.0001g of salt

Wheat gluten

Total amount of wheat gluten = 2kg
Each unit of big loaf requires 0.167g of wheat gluten
Each unit of giant loaf requires 0.25 g of wheat gluten
Each unit of small loaf requires 0.12g of wheat gluten

Soybean Oil

Total amount (volume) of soybean available = 10.0L
Each unit of big loaf requires 0.2 L of soybean oil
Each unit of giant loaf requires 0.5 L of soybean oil
Each unit of small loaf requires 0.09L of soybean oil

Profit contribution per unit product (size) of bread produced

Each unit of big loaf = N40
Each unit of giant loaf = N50
Each unit of small loaf = N10

Table 1
Profit contribution of bread loafs

Raw Materials	big loaf	giant loaf	small loaf	Total Raw Material
Flour(Kg)	0.5	1	0.34	300
Sugar(Kg)	0.4	0.5	0.3	50
Yeast(Kg)	0.2	0.2	0.2	20
Salt(Kg)	0.002	0.02	0.001	1
Wheat Gluten (Kg)	0.167	0.25	0.12	2
Soybean Oil(L)	0.2	0.5	0.09	10
Profit(N)	40	50	10	

The above data can be summarized in a tabular form.

Model formulation:

Let the quantity of big loaf to be produce = x_1
Let the quantity of giant loaf to be produce = x_2
Let the quantity of small loaf to be produce = x_3
Let Z denote the profit to be maximize

The linear programming model for the above production data is given by

MAXIMIZE: $40x_1 + 50x_2 + 10x_3$
$0.5x_1 + 1x_2 + 0.34\,x_3 \leq 300$
$0.4x_1 + 0.5x_2 + 0.3x_3 \leq 50$
$0.2x_1 + 0.2x_2 + 0.2x_3 \leq 20$
$0.002x_1 + 0.02x_2 + 0.001x_3 \leq 1$
$0.167x_1 + 0.25x_2 + 0.12x_3 \leq 2$
$0.2x_1 + 0.5x_2 + 0.09x_3 \leq 10$
$x_1 + x_2 + x_3 \geq 0$

Standard Form of given LPP is:

Maximize Z= $40x_1 + 50x_2 + 10x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9$

Subject to;
$0.5x_1 + 1x_2 + 0.34\,x_3 + 0x_4 = 300$
$0.4x_1 + 0.5x_2 + 0.3x_3 + 0x_5 = 50$
$0.2x_1 + 0.2x_2 + 0.2x_3 + 0x_6 = 20$
$0.002x_1 + 0.02x_2 + 0.001x_3 + 0x_7 = 1$
$0.167x_1 + 0.25x_2 + 0.12x_3 + 0x_8 = 2$
$0.2x_1 + 0.5x_2 + 0.09x_3 + 0x_9 = 10$
$x_1 + x_2 + \ldots\ldots + x_9 > 0$

The above equations were solved using simplex method which gave the optimal solution as;

$x_1 = 500.00$
$x_2 = 0.00$
$x_3 = 1000.00$
Z =30,000.00

5. Result

The results derived above that Based on the data collected indicates that only two sizes of bread should be produced, viz.; Small loaf and Big loaf. We can conclude that the quantities produced are 500 and 1000 units for big loaf and small loaf respectively. The small loaf is produced in greater quantities to produce a maximum profit of N30000.

6. Conclusion

The objective of this paper is produce a maximum profit of raw materials in bakery using linear programming for bread production. The decision variables are three different sizes of bread (big loaf, giant loaf and small loaf) produced by a bakery. The analysis mainly depends on collecting six raw materials that are used in the production of breads viz., flour, and sugar, yeast, salt, and wheat gluten and soybean oil size. The result shows that 1000 unit of small loaf, 500 unit of big loaf and 0 unit of giant loaf should be produce respectively which will give a maximum profit of 30000.Thus conclusion was made that small loaf breads are produced in greater quantities to achieve profit maximization. As a future work we could solve the above analysis of bakery raw materials using either of the Big-M or the two phase method

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