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Interpolat

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***Abstract*: In this paper we will come across introduction to interpolation and calculus of finite differences. It further includes various polynomial interpolation methods like that of Lagranges, Newton's forward, backward & central difference method. These help us to calculate any number of numerical integrations with minimal error. The main idea lies in increasing the coefficients rather than an interval. In order to reduce the numerical computations a formula has been derived from Newtons interpolation method. Application of this formula can seen and is formulated below.**

***Keywords*: Interpolat**

1. Introduction

The process to estimate or predict points that are unmeasured using sample points with no approximation in values. Estimation of data for points using known data within the region. It is also used to derive a function that is at its simplest state function and passes through all the points. Usually it is done through approximation of the required function taking into consideration of simpler functions as in polynomials. More accurate and discrete functions can be done through Splines and Chebicheve. In case using interpolation the value of variable dependent. Thus Newton's interpolation is applied for reconstruction of a signal to enhance reconstruction of a signal. In this section, we will get to know the polynomial interpolation in the form of Lagrange and Newton. Given a sequence of (n +1) data points and a function f, so here our motive would be to determine an n-th degree polynomial which interpolates f at these points. We thus resort to the notion of divided differences.

A. Uses

1. Estimating data points.
2. Solving discrete experimental data.
3. Simplification of complicated functions
4. Easier in evaluating differentiating and integrating.

2. Polynomial Interpolation

Interpolation is commonly used for polynomials as polynomials prove to be easier in differentiating and integrating. Polynomials serve to approximate curves with higher values. If a set of data contains n points, then there exists only one polynomial having a degree{n-1} or smaller. It is a method used for estimation of data. The highest power or exponent is termed as the degree of polynomial.

The different methods for finding out polynomial interpolation are,

1. Lagranges
2. Newton's forward method
3. Newton's backward method
4. Sterling and bessel's interpolation methods

3. Linear interpolation

Let us consider function (x0, f0), (x, f1). We need to find a function f(x) which passes through two given data points. The formula for linear interpolation is,

$$g(x) = f(x_0)\frac{(x_1-x)}{(x_1-x_0)} + f(x_1)\frac{(x-x_0)}{(x_1-x_0)}$$

Errors for Linear interpolation functions

A constant equation for an error can be expressed as

e(x)=f(x) - g(x)

4. Newton's Forward and Backward Interpolation

The method of estimating the value of a function for any intermediate value of the independent variable is called interpolation whereas the process of computation of a value outside its given range is called extrapolation. This named after Sir Isaac Newton, Newton's Interpolation proves to be a popular polynomial interpolating technique of numerical analysis and mathematics. Here, the coefficients of polynomials are formulated by using divided difference, so this method of interpolation is also known as Newton's divided difference interpolation polynomial. Newton polynomial interpolation includes 2 types. They are mainly as Newton's forward difference formula and Newton's backward difference formula.

Let us consider a problem and use Newton's forward difference method to solve.

x	0.1	0.2	0.3	0.4	0.5
y=f(x)	1.4	1.56	1.76	2	2.28

The forward difference table to the given data is,

x	y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	1.4				
0.2	1.56	0.16			
0.3	1.76	0.2	0.04		
0.4	2	0.24	0.04	0	
0.5	2.28	0.28	0.04	0	0

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0$$

$x_0 = 0.1 \quad y_0 = 1.40$
$\Delta y_0 = 0.16 \quad \Delta^2 y_0 = 0.04$

$$p = \frac{x - 0.1}{0.1} = 10x - 1$$

Substituting theses values

$$y = f(x) = 1.40 + (10x-1)(0.16) + \frac{(10x-1)(10x-2)}{2}(0.04)$$

=1.655

5. Lagranges's Interpolation method

This method is used over Newton's interpolation. Moreover, it is also applicable for unequally spaced values of x. The interpolating polynomial of the least degree is unique as it can be arrived at through multiple methods, this means that an answer can be achieved through different ways and methods. Now the question arises which method to prefer?

Thus referring to "the Lagrange polynomial" is perhaps not as correct as referring to "the Lagrange form" of the polynomial. It can also be used to find any intermediate data points. Let us consider 3 data points (x_0, y_0), (x_1, y_1), (x_2, y_2) which are evenly spaced.

Since all the 3 points are interpolated. Computing and substituting the values we can formulate a general formula.

Let us now solve a question using lagranges Interpolation method.

x	y
1	0
2	7
3	26
5	124

	$(x - x_1) .. (x - x_n)$		$(x - x_0)..(x - x_{n-1})$	
f(x) =	———	f_0+	———	f_n
	$(x_0 - x_1) .(x_0 - x_n)$		$(x_n - x_0)\dots(x_n - x_{n-1})$	

n	(	n			)f_i	
Σ		| |		$x - x_j$		
		j = 0		$(x_i - x_j)$		
i = 0		j□≠ 1				

$$\frac{(4-1)(4-2)(4-3)}{(2-1)(2-3)(2-5)} * (7) + \frac{(4-1)(4-2)(4-5)}{(3-1)(3-2)(3-5)} * (26) + \frac{(4-1)(4-2)(4-3)}{(5-1)(5-2)(5-3)} * (124)$$

$y_r = 63$

6. Conclusion

So this paper proposes us the methods of interpolation and how they are differently used in different situations to find out the solution with minimal errors. Comparison of various methods with the new formula and finding out the simpler one.

Thus this suggests us simpler ways to find the desired result. Thus as in conclusion this formula is formulated whose function will remain constant or increase with variables that are independent. As a clear conclusion can be drawn that Lagrange's method has advantages over Newton's method.

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