Some of Edge Product and Total Edge Product Cordial Labeling of Diamond Grid

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Abstract—In this paper we introduce an edge product cordial labeling graph. For a graph, G = (V(G), E(G)), a function f : E(G) → {0, 1} is called an edge product cordial labeling of G if the induced vertex labeling function defined by the product of incident edge such that the edges labeled with 1 and label 0 differ by at most 1. Similarly, the vertices labeled with 1 and label 0 also differ by at most 1.

Index Terms—Cordial Labeling, Vertices, edges, total edge product.

I. INTRODUCTION

Graphs are discrete structure which constitutes of vertices and edges that connect these vertices. The graph models can be used to represent almost every problem involving discrete arrangement of objects. They are not concerned with their internal properties but with their inter-relationship. By a graph theory, we mean a finite undirected graph without loops or multiple edges. Vaidya and Barasara [1] introduced the edge analogue of product cordial labeling and named it as edge product cordial labeling. Aisha explicits the 3-total edge product cordial labeling of rhombic grid [2]. In this paper, we analyze the edge product and total edge product cordial labeling of diamond grid.

Definition 1.1
The assignment of integers to the vertices or edges, or both, which was subjected to certain condition(s) is called graph labeling. If the domain of mapping is the set of vertices (or edges) then the labeling is called a vertex(or an edge) labeling [3].

Definition 1.2
A graph G = (V, E) with q edges to be graceful, if there is an injection φ from the vertices V(G) is denoted by the set {0,1,2,3,...,q} such that the induced function φ* is denoted by the set {1,2,3,4,...,q}, then φ*(e = xy) = |φ(x) − φ(y)| for each edge e = xy is a bijection and the resulting edge φ is said to be graceful labeling of G.

Definition 1.3
A graph G = (V, E) with q edges is said to be harmonious, if there is an injection φ from the vertices V(G) is denoted by the module q such that the induced function φ* is denoted by the set {1,2,3,4,...,q}, then φ*(e = xy) = |φ(x) + φ(y)| for each edge e = xy is a bijection and the resulting edge φ is said to be harmonious labeling of G.

Definition 1.4
For a graph G, an edge labeling function φ: V(G) → {0, 1} defined as φ(v) = Σ φ(x)/xy E(G). The function φ is called E-cordial labeling of G if |e0(0) − e1(0)| ≤ 1 and |vφ(0) − vφ(1)| ≤ 1. A graph is called E-cordial if it admits E-cordial labeling.

Definition 1.5
For a graph G, an edge labeling function φ*: E(G) → {0, 1} induces a vertex labeling function φ: V(G)→{0,1} defined as φ(v) = Σ|φ(0) + φ(1)| ≤ 1 if φ(v) = 0 or 1 or 2. The function φ is called edge product cordial labeling of G if |eφ(0) − eφ(1)| ≤ 1 and |vφ(0) − vφ(1)| ≤ 1. A graph is called edge product cordial if it admits edge product cordial labeling.

Definition 1.6
For a graph G, an edge labeling function φ*: E(G) → {0, 1} induces a vertex labeling function φ: V(G)→{0,1} defined as φ(v) = Π|φ(0) + φ(1)| ≤ 1 if φ(v) = 0 or 1 or 2. The function φ is called total product cordial labeling of G if |eφ(0) + eφ(1) − vφ(0) − vφ(1)| ≤ 1. A graph is called total product cordial if it admits total product cordial labeling.

Definition 1.7
For a graph G, a vertex labeling function φ: V(G) → {0, 1} induces an edge labeling function φ*: E(G) → {0, 1} defined as φ*(xy) = φ(x) + φ(y). The function φ is called total product cordial labeling of G if |eφ(0) + eφ(1) − vφ(0) − vφ(1)| ≤ 1. A graph is called total product cordial if it admits total product cordial labeling.

Definition 1.8
For a graph G, an edge labeling function φ*: E(G) → {0, 1} induces a vertex labeling function φ: V(G)→{0,1} defined as φ(v) = Σ|φ(0) + φ(1)| ≤ 1 if φ(v) = 0 or 1 or 2. The function φ is called total edge product cordial labeling of G, if |eφ(0) + eφ(1) − vφ(0) + vφ(1)| ≤ 1.
A graph is said to be total edge product cordial if it admits total edge product cordial labeling.

Definition: 1.9
Let us consider $C_n(\phi)$ denote the one - point union of $t$ cycles of length $m$.

Definition: 1.10
The diamond wheel $W_m$ is defined to be the join $C_m + K_t$. The vertex corresponding to $K_t$ is known as apex vertex, the vertices corresponding to cycle are known as rim vertices.

Definition: 1.11
Let $e = xy$ be an edge of graph $G$ and $w$ is not a vertex of $G$. The edge $e$ is subdivided when it is replaced by the edges $e = xw$ and $e = wx$.

Definition: 1.12
The gear graph $G_m$ is obtained from the wheel $W_m$ by subdividing each of its rim edge.

Definition: 1.13
The fan $\varphi_m$ is the graph obtained by taking $n - 2$ concurrent chords in cycle $C_{m+1}$. The vertex at which all the chords are concurrent is called the apex vertex. It is also given by $\varphi_m = P_m + K_1$.

Definition: 1.14
The double fan $D\varphi_m$ is defined as $P_m + 2K_1$.

II. MATHEMATICAL FORMULATION

Theorem: 2.1
Every edge product cordial graph of either even order or even size admit total edge product cordial labeling.

Proof:
Let $G$ be an edge product cordial graph with the order $p$ and size $q$. In order to prove, three cases are possible.

Case 1: When $p$ is odd and $q$ is even.
Since $G$ is edge product cordial graph,

\[ e_0(0) = e_0(1) = \frac{q}{2} \quad \text{and} \quad \left| v_0(0) - v_0(1) \right| = 1. \]

Therefore, \[ \left| (v_0(0) + e_0(0)) - (v_0(1) - e_0(1)) \right| = 1. \]

Case 2: When $p$ is even and $q$ is odd.
Since $G$ is edge product cordial graph,

\[ v_0(0) = v_0(1) = \frac{p}{2} \quad \text{and} \quad \left| e_0(0) - e_0(1) \right| = 1. \]

Therefore, \[ \left| (v_0(0) + e_0(0)) - (v_0(1) - e_0(1)) \right| = 1. \]

Case 3: When $p$ is even and $q$ is even.
Since $G$ is an edge product cordial graph,

\[ v_0(0) = v_0(1) = \frac{p}{2} \quad \text{and} \quad e_0(0) = e_0(1) = \frac{q}{2}, \]

Therefore, \[ \left| (v_0(0) + e_0(0)) - (v_0(1) - e_0(1)) \right| = 0. \]

Thus in either case $G$ satisfies the condition for total edge product cordial. (i.e) $G$ shows the total edge product cordial labeling.

Theorem: 2.2
The diamond grid graph with degree sequences $(1,1)$, $(2,2,2,2)$ or $(3,2,2,1)$ are not total edge product cordial graphs.

Proof:
For the graph with degree sequence $(1,1)$ has only one edge and two vertices. If we label the edge with $1$ or $0$ then both the vertices will give the same label.

As a consequence, \[ \left| (v_0(0) + e_0(0)) - (v_0(1) - e_0(1)) \right| = 3. \]

For the graph with degree sequence $(2,2,2,2)$ or $(3,2,2,1)$ has four edges and four vertices. If we assign label $0$ to any edge then two end vertices will receive label $0$ then $v_0(0) + e_0(0) = 3$. If we assign label $0$ to two incident edges then three vertices will receive label $0$ (including a common vertex and two remaining vertices) then $v_0(0) + e_0(0) = 4$.

Suppose we assign label $0$ to two non - incident edges then four end vertices will receive label $0$. Then we have $v_0(0) + e_0(0) = 6$. Then in every situations \[ \left| (v_0(0) + e_0(0)) - (v_0(1) + e_0(1)) \right| > 2. \]

Hence, the graph with degree sequences $(1,1)$, $(2,2,2,2)$ or $(3,2,2,1)$ are not total edge product cordial graphs.

Theorem: 2.3
The diamond cycle $D_m$ is a total edge product cordial graph except for $m \neq 4$.

Proof:
Let $v_1, v_2, v_3, v_4$ be the vertices of cycle $D_m$. The following two cases.

Case 1: When $m$ is odd.
\[ \varphi(v_i, v_{i+1}) = 0; \quad 1 \leq i \leq \frac{m}{2} \]
\[ \varphi(v_i, v_{i+1}) = 1; \quad \frac{m}{2} + 1 \leq i \leq m - 1 \]
\[ \varphi(v_i, v_m) = 1. \]

Case 2: When $m$ is even and $m \neq 4$.
\[ \varphi(v_i, v_{i+1}) = 0; \quad 1 \leq i \leq \frac{m-1}{2} \]
\[ \varphi(v_i, v_{i+1}) = 1; \quad \frac{m-1}{2} + 1 \leq i \leq m - 1 \]
\[ \varphi(v_i, v_m) = 1. \]

In both the cases we have $v_0(0) + e_0(0) = m$ and $v_0(1) + e_0(1) = m$. So, \[ \left| (v_0(0) + e_0(0)) - (v_0(1) - e_0(1)) \right| \leq 1. \]

Hence, the diamond cycle $D_m$ is a total edge product cordial graph except for $m \neq 4$.

Theorem: 2.4
The graph $D_m^{(\phi)}$ be the total edge product cordial graph.
Case 1: When \( t \) is even. Here \( D_{m}^{(0)} \) is of even size and it is edge product cordial graph.

Case 2: When \( t \) and \( m \) both are odd

\[
\begin{align*}
\varphi(v_{i}, v_{j+1}) &= 0; \quad 1 \leq i \leq \frac{t-1}{2} \quad \text{and} \quad 1 \leq j \leq m - 2 \\
\varphi(v_{i}, v_{j}) &= 0; \quad 1 \leq i \leq \frac{t-1}{2} \\
\varphi(v_{i}, v_{j+1}) &= 1; \quad \frac{t+1}{2} \leq i \leq t - 1 \quad \text{and} \quad 1 \leq j \leq m - 2 \\
\varphi(v_{i}, v_{j}) &= 1; \quad \frac{t+1}{2} \leq i \leq t - 1 \\
\varphi(v_{i}, v_{j+1}) &= 1; \quad 1 \leq i \leq \frac{m-3}{2} \\
\varphi(v_{i}, v_{j}) &= 0; \\
\varphi(v_{i}, v_{j+1}) &= 1; \quad \frac{m-1}{2} \leq i \leq m - 2 \\
\varphi(v_{i}, v_{j}) &= 0.
\end{align*}
\]

Case 3: When \( t \) is odd and \( m \) is even.

\[
\begin{align*}
\varphi(v_{i}, v_{j+1}) &= 0; \quad 1 \leq i \leq \frac{t-3}{2}, 1 \leq j \leq m - 2 \\
\varphi(v_{i}, v_{j}) &= 0; \quad 1 \leq i \leq \frac{t-3}{2} \\
\varphi(v_{i}, v_{j+1}) &= 0; \quad \frac{t+3}{2} \leq i \leq t - 1, 1 \leq j \leq m - 2 \\
\varphi(v_{i}, v_{j}) &= 1; \quad \frac{t+3}{2} \leq i \leq t - 1 \\
\varphi(v_{i}, v_{j+1}) &= 0; \quad i = \frac{m+1}{2} \leq j \leq m - 2 \\
\varphi(v_{i}, v_{j}) &= 1; \quad i = \frac{m+1}{2} \\
\varphi(v_{i}, v_{j+1}) &= 1; \quad \frac{m+1}{2} \leq i \leq t \\
\varphi(v_{i}, v_{j}) &= 1; \quad \frac{m+1}{2} \leq i \leq t \\
\varphi(v_{i}, v_{j+1}) &= 1; \quad \frac{m+1}{2} \leq i \leq t \\
\varphi(v_{i}, v_{j}) &= 1; \quad \frac{m+1}{2} \leq i \leq t \\
\end{align*}
\]

In case 2 and case 3 we have \( v_{0}(0) + e_{0}(0) = \frac{2mt-t+1}{2} \) and \( v_{0}(1) + e(1) = \frac{2mt-t-1}{2} \). Therefore \( \left| (v_{0}(0) + e_{0}(0)) - (v_{0}(1) + e(1)) \right| \leq 1 \). Hence, the graph \( D_{m}^{(0)} \) is a total edge product cordial graph.

Example:

Fig. 1. Diamond wheel graph