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Some of Edge Product and Total Edge Product Cordial Labeling of Diamond Grid

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***Abstract*—In this paper we introduce an edge product cordial labeling graph. For a graph, G = (V(G) , E(G)), a function f : E(G)→ {0 , 1} is called an edge product cordial labeling of G if the induced vertex labeling function defined by the product of incident edge such that the edges labeled with 1 and label 0 differ by at most 1. Similarly, the vertices labeled with 1 and label 0 also differ by at most 1.**

***Index Terms*—Cordial Labeling, Vertices, edges, total edge product.**

I. INTRODUCTION

Graphs are discrete structure which constitutes of vertices and edges that connect these vertices. The graph models can be used to represent almost every problem involving discrete arrangement of objects. They are not concerned with their internal properties but with their inter-relationship. By a graph theory, we mean a finite undirected graph without loops or multiple edges. Vaidya and Barasara [1] introduced the edge analogue of product cordial labeling and named it as edge product cordial labeling. Aisha explicits the 3-total edge product cordial labeling of rhombic grid [2].In this paper, we analysis the edge product and total edge product cordial labeling of diamond grid

Definition: 1.1
The assignment of integers to the *vertices* or *edges,* or both, which was subjected to certain condition(s) is called graph labeling. If the domain of mapping is the set of vertices (or edges) then the labeling is called a *vertex(or an edge)* labeling [3].

Definition: 1.2
A graph $G = (V,E)$ with q edges to be *graceful,* if there is an injection φ from the vertices $V(G)$ is denoted by the set $\{0,1,2,,....q\}$ such that the induced function φ^* is denoted by the set $\{1,2,3,4........q\}$, then $\varphi^*(e = xy)= | \varphi(x) - \varphi(y) |$ for each edge $e = xy$ is a bijection and 'φ' is called *graceful labelling* of G.

Definition: 1.3
A graph $G = (V, E)$ with q edges is said to be harmonious, if there is an injection φ from the vertices $V(G)$ is denoted by the module q such that the induced function φ^* is denoted by the set $\{1,2,3,4........q\}$, then $\varphi^*(e = xy) = | \varphi(x) + \varphi(y) |$ for each edge $e = xy$ is a bijection and the resulting edge φ is said to be *harmonious labeling* of *G.*

Definition: 1.4
For a graph G, an edge labeling function $\varphi: V(G)$ is denoted by the set$\{0,1\}$ which induces by the vertex labeling function $\varphi^*: E(G)$ denotes $\{0,1\}$ defined as $\varphi^*(xy) = | \varphi(x) - \varphi(y)$.
Let us consider the number of vertices of G labeled with i under φ be $v_\varphi(i)$ and the number of edges G labeled under φ^* be $e_\varphi(i)$ for $i = 0,1$. This function φ is said to be *cordial labeling of* G.
Since $| e_\varphi(0) - e_\varphi(1) | \leq 1$ and $| v_\varphi(0) - v_\varphi(1) | \leq 1$. A graph is said to be cordial labeling.

Definition: 1.5
For a graph G, an edge labeling function $\varphi^*: E(G) \rightarrow \{0,1\}$ induces a vertex labeling function $\varphi: V(G)\rightarrow \{0,1\}$ defined as $\varphi(v)= \sum\{ \varphi^*(xy) / xy \in E(G)\}$. The function φ^* is called E – *cordial labeling* of G if $| e_\varphi(0) - e_\varphi(1) | \leq 1$ and $| v_\varphi(0) - v_\varphi(1) | \leq 1$. A graph is called E – cordial if it admits E –*cordial labeling.*

Definition: 1.6
For a graph G, an edge labeling function $\varphi^*: E(G) \rightarrow \{0,1\}$ induces a vertex labeling function $\varphi: V(G) \rightarrow \{0,1\}$defined as $\varphi(v) = \prod \{\varphi^*(xy)/ xy \in E(G) \}$. The function φ^* is called *edge product cordial labeling* of G if$| e_\varphi(0) - e_\varphi(1) | \leq 1$ and $| v_\varphi(0) - v_\varphi(1) | \leq 1$. A graph is called *edge product cordial* if it admits *edge product cordial labeling.*

Definition: 1.7
For a graph G, a vertex labeling function $\varphi: V(G) \rightarrow \{0,1\}$ induces an edge labeling function $\varphi^*: E(G) \rightarrow \{0,1\}$defined as $\varphi^*(xy) = \varphi(x)\, \varphi(y)$. The function φ is called *total product cordial labeling* of G if $| (v_\varphi(0) + e_\varphi(0)) - (v_\varphi(1) + e_\varphi(1)) | \leq 1$. A graph is called *total product cordial* if it admits *total product cordial labeling.*

Definition: 1.8
For a graph G, an edge labeling function $\varphi^*: E(G) \rightarrow \{0,1\}$ induces a vertex labeling function $\varphi :V(G) \rightarrow \{0,1\}$ defined as $\varphi(v) = \prod \{ \varphi^*(xy) / xy \in E(G)\}$. The function φ^* is called a *total edge product cordial labeling* of G,
if $| (v_\varphi(0) + e_\varphi(0)) - (v_\varphi(1) + e_\varphi(1) | \leq 1$.

A graph is said to be *total edge product cordial* if it admits *total edge product cordial labeling.*

Definition: 1.9
Let us consider $C_m^{(t)}$ denote the one – point union of t *cycles* of length m.

Definition: 1.10
The *diamond wheel* W_m is defined to be the join $C_m + K_1$. The vertex corresponding to K_1 is known as *apex vertex*, the vertices corresponding to cycle are known as *rim vertices.*

Definition: 1.11
Let $e = xy$ be an edge of graph G and w is not a vertex of G. The edge e is subdivided when it is replaced by the edges $e' = xw$ and $e'' = wx$.

Definition: 1.12
The *gear graph* G_m is obtained from the wheel W_m by subdividing each of its rim edge.

Definition: 1.13
The fan φ_m is the graph obtained by taking $n – 2$ concurrent chords in cycle C_{m+1}. The vertex at which all the chords are concurrent is called the *apex vertex*. It is also given by $\varphi_m = P_m + K_1$.

Definition: 1.14
The *double fan* $D\varphi_m$ is defined as $P_m + 2K_1$.

II. MATHEMATICAL FORMULATION

Theorem: 2.1
Every edge product cordial graph of either even order or even size admit total edge product cordial labeling.

Proof:
Let G be an edge product cordial graph with the order p and size q. In order to prove, three cases are possible.

Case 1: When p is odd and q is even.
Since G is edge product cordial graph,

$$e_\varphi(0) = e_\varphi(1) = \frac{q}{2} \quad \text{and} \quad |v_\varphi(0) - v_\varphi(1)| = 1.$$

Therefore, $|(v_\varphi(0)+e_\varphi(0)) – (v_\varphi(1) – e_\varphi(1))| = 1$.

Case 2: When p is even and q is odd.
Since G is edge product cordial graph,

$$v_\varphi(0) = v_\varphi(1) = \frac{p}{2} \quad \text{and} \quad |e_\varphi(0) - e_\varphi(1)| = 1.$$

Therefore, $|(v_\varphi(0)+e_\varphi(0)) – (v_\varphi(1) – e_\varphi(1))| = 1$.

Case 3: When p is even and q is even.
Since G is an edge product cordial graph,

$$v_\varphi(0) = v_\varphi(1) = \frac{p}{2} \text{ and}$$
$$e_\varphi(0) = e_\varphi(1) = \frac{q}{2}.$$

Therefore, $|(v_\varphi(0)+e_\varphi(0)) – (v_\varphi(1) – e_\varphi(1))| = 0$.

Thus in either case G satisfies the condition for total edge product cordial. (i.e) G shows the total edge product cordial labeling.

Theorem: 2.2
The diamond grid graph with degree sequences (1,1), (2,2,2,2) or (3,2,2,1) are not total edge product cordial graphs.

Proof:
For the graph with degree sequence (1, 1) has only one edge and two vertices. If we label the edge with 1 or 0 then both the vertices will give the same label.

As a consequence, $|(v_\varphi(0) + e_\varphi(0)) – (v_\varphi(1) – e_\varphi(1))| = 3$.
For the graph with degree sequence (2,2,2,2) or (3,2,2,1) has four edges and four vertices. If we assign label 0 to any edge then two end vertices will receive label 0 then $v_\varphi(0) + e_\varphi(0) = 3$. If we assign label 0 to two incident edges then three vertices will receive label 0 (including a common vertex and two remaining vertices) then $v_\varphi(0) + e_\varphi(0) = 4$. Suppose we assign label 0 to two non - incident edges then four end vertices will receive label 0. then $v_\varphi(0) + e_\varphi(0) = 6$. Then in every situations $|(v_\varphi(0) + e_\varphi(0)) – (v_\varphi(1) + e_\varphi(1))| > 2$. Hence, the graph with degree sequences (1,1), (2,2,2,2) or (3,2,2,1) are not total edge product cordial graphs.

Theorem: 2.3
The diamond cycle D_m is a total edge product cordial graph except for $m \neq 4$.

Proof:
Let $v_1, v_2, v_3, v_4, \ldots\ldots v_m$ be the vertices of cycle D_m.
The following two cases.

Case 1: When m is odd.

$$\varphi(v_i\ v_{i+1}) = 0; \quad 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor$$
$$\varphi(v_i\ v_{i+1}) = 1; \quad \left\lceil \frac{m}{2} \right\rceil \le i \le m - 1$$
$$\varphi(v_1\ v_m) = 1.$$

Case 2: When m is even and $m \nmid 4$.

$$\varphi(v_i\ v_{i+1}) = 0; \quad 1 \le i \le \frac{m-4}{2}$$
$$\varphi(v_i\ v_{i+1}) = 1; \quad i = \frac{m-2}{2}$$
$$\varphi(v_i\ v_{i+1}) = 0; \quad i = \frac{m}{2}$$
$$\varphi(v_i\ v_{i+1}) = 1; \quad \frac{m}{2} + 1 \le i \le m-1$$
$$\varphi(v_1\ v_m) = 1.$$

In both the cases we have $v_f(0) + e_f(0) = m$ and $v_\varphi(1) + e_\varphi(1) = m$. So, $|(v_\varphi(0) + e_\varphi(0)) – (v_\varphi(1) – e_\varphi(1))| \le 1$.
Hence, the diamond cycle D_m is a total edge product cordial graph except for $m \neq 4$.

Theorem: 2.4
The graph $D_m^{(t)}$ be the total edge product cordial graph.

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Proof:

Let $v_{k,1}$ $v_{k,2}$ ....$v_{k,m-1}$ be the vertices of k^{th} copy of the cycle D_m and v be the common vertex of $D_m^{(t)}$. The vertices vk, 1 and $v_{k,m-1}$ of k^{th} copy of cycle D_m are adjacent to v.

$|V(D_m^{(t)})| = t(m-1)+1$ and $|E(D_m^{(t)})| = tm$.

Case 1: When t is even.

Here $D_m^{(t)}$ is of even size and it is edge product cordial graph.

Case 2: When t and m both are odd

$\varphi(v_{i,j},v_{i,j+1}) = 0;\quad 1 \le i \le \frac{t-1}{2}$ and $1 \le j \le m - 2$

$\varphi(vv_{i,m-1}) = 0;\quad 1 \le i \le \frac{t-1}{2}$

$\varphi(vv_{i,m-1}) = 0;\quad 1 \le i \le \frac{t-1}{2}$

$\varphi(v_{i,j}v_{i,j+1}) = 1;\quad \frac{t+1}{2} \le i \le t-1$ and $1 \le j \le m-2$

$\varphi(vv_{i,1}) = 1;\quad \frac{t+1}{2} \le i \le t-1$

$\varphi(vv_{i,n-1}) = 1;\quad \frac{t+1}{2} \le i \le t-1$

$\varphi(v_{t,i}v_{t,i+1}) = 0;\quad 1 \le i \le \frac{m-3}{2}$

$\varphi(vv_{t,1}) = 0;$

$\varphi(v_{t,i}v_{t,i+1}) = 1;\quad \frac{m-1}{2} \le i \le m - 2$

$\varphi(vv_{t,m-1}) = 1.$

Case 3: When t is odd and m is even.

$\varphi(v_{i,j},v_{i,j+1}) = 0;\quad 1 \le i \le \frac{t-3}{2}, 1 \le j \le m-2$

$\varphi(vv_{i,1}) = 0;\quad 1 \le i \le \frac{t-3}{2}$

$\varphi(vv_{i,m-1}) = 0;\quad 1 \le i \le \frac{t-3}{2}$

$\varphi(v_{i,j}v_{i,j+1}) = 0;\quad \frac{t+1}{2} \le i \le t-1, 1 \le j \le m-2$

$\varphi(vv_{i,m-1}) = 1;\quad i = \frac{t-1}{2}$

$\varphi(v_{i,j}v_{i,j+1}) = 0;\quad i = \frac{t+1}{2}, 1 \le j \le \frac{m-2}{2}$

$\varphi(vv_{i,1}) = 0;\quad i = \frac{t+1}{2}$

$\varphi(v_{i,j}v_{i,j+1}) = 1;\quad i = \frac{t+1}{2}, \frac{m}{2} \le j \le m-2$

$\varphi(vv_{i,m-1}) = 1;\quad i = \frac{t+1}{2}$

$\varphi(vv_{i,1}) = 1;\quad \frac{t+3}{2} \le i \le t$

$\varphi(v_{i,j}v_{i,j+1}) = 1;\quad \frac{t+3}{2} \le i \le t, 1 \le j \le m-2$

$\varphi(vv_{i,m-1}) = 1;\quad \frac{t+3}{2} \le i \le t$

In case 2 and case 3 we have $v_\varphi(0) + e_\varphi(0) = \frac{2mt-t+1}{2}$ and $v_\varphi(1) + e(1) = \frac{2mt-t+1}{2}$. Therefore $|(v_\varphi(0) + e_\varphi(0)) - (v_\varphi(1) + e_\varphi(1)| \le 1$. Hence, the graph $D_m^{(t)}$ is a total edge product cordial graph.

Example:

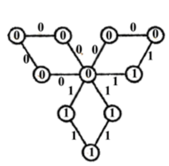

Fig. 1. Diamond wheel graph

The diamond wheel graph $D_4^{(3)}$ and its total edge product cordial labeling.

Theorem: 2.6

The diamond shape D_m is a total edge product cordial graph.

Proof:

Let $v_1,v_2,v_3,$...............,v_m be the rim vertices and v be an apex vertex of wheel D_m. To define φ: $E(D_m)$ denotes the set{0,1} we will consider following two cases.

Case 1: When m is even.

$\varphi(v_{2i-1}v_{2i}) = 0;\quad 1 \le i \le \frac{m}{2}$

$\varphi(v_1v_n) = 1;$

$\varphi(v_{2i}v_{2i+1}) = 1;\quad 1 \le i \le \frac{m-2}{2}$

$\varphi(vv_i) = 1;\quad 1 \le i \le m.$

We have

$v_\varphi(0) + e_\varphi(0) = \frac{3m}{2}$ and $v_\varphi(1) + e_\varphi(1) = \frac{3m}{2}+1.$

*Case 2***:** When m is odd.

$\varphi(v_{2i-1}v_{2i}) = 0;\quad 1 \le i \le \frac{m-1}{2}$

$\varphi(v_1v_m) = 0;$

$\varphi(v_{2i}v_{2i+1}) = 1;\quad 1 \le i \le \frac{m-1}{2}$

$\varphi(vv_i) = 1;\quad 1 \le i \le m.$

We have $v_\varphi(0)+e_\varphi(0) = \frac{3m+1}{2}$ and

$e_\varphi(1)+e_\varphi(1) = \frac{3m+1}{2}$

For both the cases, we have

$|(v_\varphi(0)+e_\varphi(0)) - (v_\varphi(1)+e_\varphi(1))| \le 1.$

Hence, the diamond grid D_m is a total edge product cordial graph.

III. Conclusion

Labeling of discrete structure is a potential area of research. We have discussed the total edge product cordial labeling for diamond related graph and derive several results on it. Further to investigate the results for various graphs as well as in the context of different graph labeling problems is an open area of research.

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