# Minimum Conditional BER Probability Receiver for Adaptive Equalizer based SC-FDMA

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Abstract- Single carrier frequency division multiple access (SCFDMA), a modified form of Orthogonal FDMA (OFDMA), is a promising technique for high data rate uplink communications in 4G mobile standards (LTE and LTE-A). Networks will be illustrated by a large number of small cells exploiting Multi-User Multiple Input - Multiple Output (MU-MIMO) in order to improve spectral efficiency and user capacity. The joint detection of multiple signals coming from users belonging to different cells is still an open challenge. To overcome this trouble in cellular systems the proposed system contains SCFDMA system with LMS based MMSE and min. conditional equalizer with different diversity gain of MIMO, for various level of multipath and also over different modulations instead of MMSE. The main objective of this paper is to design distributed sub carrier mapping for SCFDMA and analyze its metrics over localized methods. A major advantage of SC-FDMA is the peakto-average power ratio (PAPR), which is lower than that of OFDMA. SCFDMA is currently a strong candidate for the uplink multiple access scheme in the Long Term Evolution of cellular systems under consideration by the Third Generation Partnership Project (3GPP). The PAPR performance of IFDMA is better than that of LFDMA. LTE is power efficient than WiMAX. Therefore the proposed system is effective and reliable.

Index terms – SCFDMA, LTE, LMS and RLS Equalizers, sub carrier mapping.

### I. INTRODUCTION

Uplink transmission in the long term evolution (LTE) of the UMTS (universal Mobile telecommunications system) terrestrial radio access system (UTRA LTE) has numerous physical layer advances in comparison to UTRA WCDMA (wideband code division multiple access) mainly to achieve two to three times better spectral efficiency. The challenging for the uplink is that these enhancements are to be achieved, preferably, with reduced power consumption to extend the battery life and cell coverage. The radio access technique is one of the key issues in the LTE uplink air interface. In LTE, OFDMA has been selected as the multiple access schemes for downlink and SC-FDMA has been selected for uplink.

OFDM is an attractive modulation technique in a cellular environment to combat a frequency selective fading channel with a relatively low-complexity receiver. OFDM requires an expensive and inherently inefficient power amplifier in the transmitter due to the high peak-to-average power ratio (PAPR) of the multicarrier signal. An alternative transmission scheme with the same attractive multipath interference mitigation property as OFDM is SC-CP (single-carrier transmission with cyclic prefix). Therefore, SC-CP can achieve a link-level performance comparable to OFDM for the same complexity, but at reduced PAPR. In addition, the performance of SC-CP can be further improved by using a turbo equalization receiver.

The choice of single-carrier transmission in a discrete Fourier transform (DFT)-spread OFDM form allows for a relatively high degree of commonality with the downlink OFDM scheme and the possibility to use the same system parameters. Moreover, it enables direct access to the frequency domain to perform frequency domain equalization.

#### 1.1 SC-FDMA Transmission

The basic uplink transmission technique is single-carrier transmission with cyclic prefix to achieve uplink interuser orthogonality and to enable efficient frequency domain equalization at the receiver side.

The transmitter and receiver structure for SC-FDMA transmission. An SC-FDMA structure is identified by the insertion of DFT spreading and inverse discrete Fourier transform (IDFT) despreading at the transmitter and receiver, respectively.

From this implementation structure, SC-FDMA is also known as DFT-spread (DFT-S) OFDM, which is a form of the single-carrier transmission technique where the signal is generated in the frequency domain. The DFT spreading combines parallel M-PSK/M-QAM symbols to form an SC-FDMA symbol. To formulate the DFT-S system, we can start by defining *Sm* as the *m*th transmitted symbol at the output of an equivalent OFDM system.

The modulator converts the random bit stream input to the M-QAM/M-PSK symbols represented by the vector Xm of length  $N_d$ . The OFDM system then constructs *Sm* as

$$Sm = Fm \times Xm \tag{1}$$

The block diagram shown Fig.1 explains transmitter side signal transmission. The input signal is given to the PSK modulator. Then modulated signal is converted as parallel signals from serial signal. These time domain signals are changed to frequency domain signals using DFT conversion. The frequency domains signals are distributive mapped then the signal are again changed to time domain getting back parallel signal to serial signal. Finally cyclic prefix is inserted in digital signal then it is transformed to analog signal and transmitted. In receiver side the transmitted signal is received and reverse process is taken. Here cyclic prefix is removed then serial signal is converted to parallel signals and time domain signal is changed to frequency domain signals, these signals are again subcarrier demapped. Finally the demapped signals are demodulated using PSK demodulator. Thus the

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transmitted signals are recovered back.

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Fig. 1. OFDMA and SC-FDMA basic transmitter and receiver structures.

#### 1.2 Peak-to-Average Power Ratio Evaluation

The envelope variation of the transmitted signal can be represented in a PAPR. In this work, the PAPR of both SC-FDMA and OFDMA is evaluated. The PAPR is an essential parameter that can affect the radio frame (RF) part because a transmitter signal with high PAPR is sensitive to the nonlinear distortion at the power amplifier. The PAPR of the transmitted signal can be expressed as

$$PAPR = \max\{|\mathbf{x}(t)|^2\} / E\{|\mathbf{x}(t)|^2\}, 0 < t < T_u.$$
(2)

Where x(t) is the instantaneous power of the signal, vi is the  $i_{th}$  signal samples,  $P_{avg}$  is the long-term average power, and PT is the power threshold of the signal. For simplicity,  $P_{avg}$  has been normalized to one. In order to obtain accurate results, the transmitted signal has been oversampled four times according to the suggestion given in Han and Lee.

The PAPR results vary depending on the modulation scheme, so higher order modulation schemes exhibit a higher PAPR compared to lower-order modulation schemes. In SC-FDMA, as a single carrier transmission, the PAPR is mainly affected by the envelope variations in each modulation scheme; thus, the higher order modulations lead to larger dynamic range and higher PAPR.

In comparison, the PAPR results for OFDMA with various modulation schemes it is observed that the PAPR is not affected by the type of modulation scheme, and all the modulation schemes have the same PAPR results. After the IFFT modulation in an OFDM system, the modulated symbols are added together to form a multicarrier signal.

## II. PROPOSED SYSTEM

In order to support more than one simultaneous transmission, multiple access techniques are employed. Both

of the techniques that will be analyzed in this project achieve are forms of Frequency Division Multiple Access (FDMA). The basic premise of converting modulation schemes such as OFDM and SC-FDE to their FDMA counterparts.

OFDMA and SC-FDMA is assigning each user a subset of the available subcarriers and having the users transmit only on at these frequencies. In this section capacity and PAPR will be investigated

#### 2.1 SCFDMA

SC-FDMA is the multiple access version of Single-Carrier Frequency-Domain Equalization (SC-FDE), which is similar to OFDM, in that they both perform channel estimation and equalization in the frequency domain. The system model for this access method. Multiple access is achieved in frequency domain in SC-FDMA. Thus to transition from SC-FDE to SC-FDMA requires division frequency amongst frequencies. This is achieved by the first performing an *N*-point FFT on the output of the symbol mapper, and mapping the output of the FFT to *N* of *M* subcarriers, with M = QN where *Q* is an integer called the bandwidth expansion factor of the symbol, which is effectively the number of Simultaneous users that the system supports.

Afterwards the *M*-point IFFT converts the signal back into time domain. Likewise in the demodulator, only *N* of the *M* frequencies of the output of the FFT correspond to this user, so those *N* values are equalized and an *N*-point IFFT is performed to get the original signal back. An important consideration to be made is how the *N* points of the signal are mapped to the *M* subcarriers of the system. Two main strategies exist: use *N* adjacent subcarriers, called localized FDMA (LFDMA) or distribute the *N* values across the *M* subcarriers using every  $Q^{\text{th}}$  subcarrier, called interleaved FDMA (IFDMA). The LTE specification follows LFDMA as it assigns each user a set of 12 adjacent subcarriers.

## 2.2 Single Carrier Modulation

Based on SC-FDMA's structure, the reasons for some of its names, such as DFT-pre coded OFDM or DFT-spread OFDM, are clear. But for the use of 'Single Carrier' in its name, SCFDMA, is not as obvious and is often the reason why is not explained, Unlike the standard OFDM where the each data symbol is carried by the individual subcarriers, the SC-FDMA transmitter carries data symbols over a group of subcarriers transmitted simultaneously.

In other words, the group of subcarriers that carry each data symbol can be viewed as one frequency band carrying data sequentially in a standard FDMA. For some of the subcarrier mappings, the time domain representation of the IFFT output will give more insight on the SC-FDMA signal. It can be mathematically shown that the SC-FDMA baseband time domain samples after IDFT or IFFT is the original data symbol set repeated in time domain over the symbol period.

As SC-FDMA modulated signal can be viewed as a single carrier signal, a pulse shaping filter can be applied to transmit signal to further improve PAPR. PAPR comparison between OFDM and SC-FDMA variations such as interleaved SC-FDMA and localized SC-FDMA has been done. With no pulse shaping filters, interleaved- SC-FDMA shows the best www.ijresm.com

PAPR. SC-FDMA offers similar performance and complexity as OFDM. However, the main advantage of SC-FDMA is the low PAPR of the transmit signal.



Fig. 2. Time domain representation of Interleaved SC-FDMA

## 2.3 SC-FDMA Modulation

SC-FDMA is a new multiple access technique that utilizes single carrier modulation, DFT spread orthogonal frequency multiplexing, and frequency domain equalization. It has a similar structure and performance as OFDM. SC-FDMA is currently adopted as the uplink multiple access scheme for 3GPP LTE. Transmitter and receiver structure for SC-FDMA and OFDM are given in Figures. It is evident from the figures that SC-FDMA transceiver has similar structure as a typical OFDM system except the addition of a new DFT block before subcarrier mapping. Hence, SC-FDMA can be considered as an OFDM system with a DFT map per.



Fig.3. OFDM Transmitter and Receiver

## 2.4 Channel-Dependent Scheduling

In general, the same modulation format is used in all the subcarriers to keep the control information overhead small. However, it is possible to have different modulation formats over multiple subcarriers, and it is in fact advantageous in harsh and time varying channel conditions. In a broadband system, the channel is frequency selective over its large system bandwidth, meaning the signal fading on each subcarrier is independent.

The interference level on each subcarrier can also be different and vary uniquely with time. It results in a different signal-to-impairment level on each of the subcarriers. Hence, having an appropriate modulation format on these subcarriers would help to maximize the overall system throughput. However, it is possible to have different modulation formats over multiple subcarriers, and it is in fact advantageous in harsh and time varying channel conditions. OFDM system inherits an adaptation of modulation formats to each of the subcarriers depending on channel conditions, and this is called Channel-dependent scheduling.

#### 2.5 Localized Mapping Vs Distributed Mapping

In localized mapping the DFT outputs are mapped to a subset of consecutive sub-carriers thereby confining them to only a fraction of the system bandwidth. In distributed mapping, the DFT outputs of the input data are assigned to subcarriers over the entire bandwidth non-continuously, resulting in zero amplitude for the remaining subcarriers. A special case of distributed SC-FDMA is called interleaved SC-FDMA, where the occupied subcarriers are equally spaced over the entire bandwidth. Figure is a general picture of localized and distributed mapping.

An example of subcarrier mapping is shown in Figure. This example assumes three users sharing 12 subcarriers. Each user has a block of four data symbols to transmit at a time. The DFT output of the data block has four complex frequency domain samples, which are mapped over 12 subcarriers using different mapping schemes. SC-FDMA inherently offers frequency diversity gain over the standard OFDM, as all information data is spread over multiple subcarriers by the DFT map per.

However, the distributed SC-FDMA is more robust with respect to frequency selective fading and offers additional frequency diversity gain, since the information is spread across the entire system bandwidth. Localized SC-FDMA in combination with channel-dependant scheduling can potentially offer multi-user diversity in frequency selective channel conditions.



Fig.4. Subcarrier Mapping

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## 2.6 LTE

The chain to generate an OFDM signal starts by paralyzing the symbols that need to be transmitted, after they are modulated. (QPSK, 16AQM, 64QAM).

#### 2.7 MMSE

Highly frequency selective channels characterized by long impulse response, ML-MUD for SC-FDMA are impractical. To counter such problem, a sub-optimal ML-MMSE detector is proposed. The proposed ML-MMSE detector removes the channel impairments and MAI using MMSE equalizer whose soft output is then fed into ML detector. Such sub-optimal MMSE-based detectors aim at maximizing Signal-to-Noise plus Interference ratio (SINR) at the detector's output.

## III. ALGORITHM

The LMS algorithm is a type of adaptive filter known as stochastic gradient-based algorithms as it utilizes the gradient vector of the filter tap weights to converge on the optimal wiener solution. It is well known and widely used due to its computational simplicity. It is this simplicity that has made it the benchmark against which all other adaptive filtering algorithms are judged.

With each iteration of the LMS algorithm, the filter tap weights of the adaptive filter are updated according to the following formula

$$w(n=1) = w(n) 2e(n)x(n)$$
 (3)

Here, x(n), is the input vector of time delayed input values, are taken as

$$\mathbf{x}(n) = [\mathbf{x}(n) \ \mathbf{x}(n-1) \ \mathbf{x}(n-2) \ \dots \ \mathbf{x}(n-N+1)]^{\mathrm{T}}$$
(4)

Vector  $\boldsymbol{w}(n){=}[w_0(n)\;w_1(n)\;w_2(n)\;...\;w_{N{\text{-}}1}(n)]^T$  represents the coefficients of the adaptive FIR filter tap weight vector at time n. The parameter  $\mu$  is known as the step size parameter and is a small positive constant. This step size parameter controls the influence of the updating factor. Selection of a suitable value for is imperative to the performance of the LMS algorithm, if the value is too small the time the adaptive filter takes to converge on the optimal solution will be too long; if  $\mu$  is too large the adaptive filter becomes unstable and its output diverges. Both the LMS and the NLMS algorithms have a fixed step size value for every tap weight in each iteration. In the Variable Step Size Least Mean Square (RLS) algorithm the step size for each iteration is expressed as a vector, w(n). Each element of the vector w(n) is a different step size value corresponding to an element of the filter tap weight vector, w(n).

#### 3.1 Derivation of the LMS Algorithm

The derivation of the LMS algorithm builds upon the theory of the wiener solution for the optimal filter tap weights,  $\mathbf{w}_0$ . It also depends on the steepest- descent algorithm as stated in equation 6, this is a formula which updates the filter coefficients using the current tap weight

vector and the current gradient of the cost function with respect to the filter tap weight coefficient vector(n).  $w(n_1) = w(n) x(n)$  (5)

As the negative gradient vector points in the direction of steepest descent for the N- dimensional quadratic cost function, each recursion shifts the value of the filter coefficients closer toward their optimum value, which corresponds to the minimum achievable value of the cost function, w(n).

The LMS algorithm is a random process implementation of the steepest descent algorithm, from equation. Here the expectation for the error signal is not known so the instantaneous value is used as an estimate. The steepest descent algorithm then becomes equation.

$$\mathbf{w}(n_1) = \mathbf{w}(n)\mathbf{x}(n) \tag{6}$$

where 
$$w(n) = e^2(n)$$
 (7)

The gradient of the cost function, w(n), can alternatively be expressed in the following form

$$W(n) = w(e(n)w^2e(n)\mathbf{x}(n))$$
(8)

Substituting this into the steepest descent algorithm

## 3.2 Implementation of the LMS Algorithm

Each iteration of the LMS algorithm requires 3 distinct steps in this order:

1. The output of the FIR filter, y(n) is calculated using equation N=1

$$\mathbf{y}(\mathbf{n}) = \mathbf{w}(\mathbf{n})\mathbf{x}(\mathbf{n}\mathbf{i})\mathbf{w}^{\mathrm{T}}(\mathbf{n})\mathbf{x}(\mathbf{n})$$
(9)

2. The value of the error estimation is calculated using equation

$$E(n) = d(n) - y(n)$$
 (10)

3. The tap weights of the FIR vector are updated in preparation for the next iteration, by equation 5.

$$\mathbf{w}(\mathbf{n}1)\mathbf{w}(\mathbf{n}) = 2\mathbf{e}(\mathbf{n})\mathbf{x}(\mathbf{n}) \tag{11}$$

The main reason for the LMS algorithms popularity in adaptive filtering is its computational simplicity, making it easier to implement than all other commonly used adaptive algorithms. For each iteration the LMS algorithm requires 2N additions and 2N+1 multiplications (N for calculating the output, y(n), one for 2e(n) and an additional N for the scalar by vector multiplication)

We first adopt a matrix formulation of Equation and collect the channel attenuations of one SCFDMA symbol (the Fourier transform of h(t) evaluated at the frequencies  $f_t$ ) in the vector h. The observed symbols after the receiver DFT become

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{n} \tag{12}$$

Where the diagonal matrix X contains the transmitted symbols on its diagonal (either known pilot symbols or receiver decisions of information symbols which we in the following assume are correct), and the vector Y contains the observed outputs of the DFT.

In this matrix notation the least-squares (LS) channel estimate (minimizing  $y - xh^2$  for all possible) becomes h

$$\mathbf{h}_{\text{LS}} = \mathbf{X}^{-1} \mathbf{y} = [\mathbf{y}_0 / \mathbf{x}_0 \ \mathbf{y}_1 / \mathbf{x}_1 \ \dots \ \mathbf{y}_{N-1} / \mathbf{x}_{N-1}]^{\text{T}}$$
(13)

This estimator simply divides the received symbol on each subcarrier by the transmitted symbol to obtain an estimate of the channel attenuation. The frequency correlation can now be used to smooth and improve the LS channel estimate. Various strategies can be adopted to use the frequency correlation.

The optimal linear minimum mean-squared error (LMMSE) estimate of h (minimizing  $E\{h - h^2\}$  for all possible linear estimators **h**) becomes

$$\mathbf{h}_{\mathrm{MMSE}} = \mathbf{A}\mathbf{h}_{\mathrm{LS}} \tag{14}$$

Where

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$$A = R_{hhLS}R^{-1}{}_{hLS} = R_{hh}(R_{hh} + \sigma_n{}^2(xx^H)^{-1})^{-1}$$
(15)

$$\mathbf{R}_{\mathbf{h}\mathbf{h}} = \mathbf{E}\{\mathbf{h}\mathbf{h}^{\mathrm{H}}\}\tag{16}$$

is the channel autocorrelation matrix, that is, the matrix containing the correlations of the channel attenuations of the subcarriers Similarly,  $R_{hhLs}$  denotes the correlation matrix between the channel attenuations and their LS-estimates, and  $R_{hhLs}$  denotes the auto correlation matrix of the LS estimates.

This LMMSE estimator is, for complexity reasons, of little practical value. The channel correlation and the SNR, it also requires N multiplications per estimated attenuation, and the dependency on the pilots or decisions X may require frequent recalculation of the matrix A. However, the LMMSE estimator (or any other high-performance and complex estimator) can be used as a basis for the design of more feasible estimators.

#### 3.3 Implementation of RLS Algorithm

RLS algorithm is executed by following these steps for each iteration. With  $\rho$ =1, each iteration of the RLS algorithm requires 4N+1 multiplication operations The output of the adaptive filter is calculated.

$$y(n) = w(n)x(n-i) w^{l}(n)x(n)$$
 (17)

The error signal is calculated as the difference between the desired output and the filter output.

$$e(n) - d(n) = y(n) \tag{18}$$

The gradient, step size and filter tap weight vectors are updated using the following equations in preparation for the next iteration.

$$g_i(n) = e(n)x(n-i)$$
; For  $i = 0, 1, ..., N-1$  (19)

$$\mathbf{g}(n) = e(n)\mathbf{x}(n), \ \mathbf{g}(n) \ \mathbf{w}_i \ (n-1) \ g_i \ (n) = g_i \ (n-1)$$
(20)

if 
$$w_i(n)_{\max} = w_i(n)$$
 else  $w_i(n)_{\min} = w_i(n)$  (21)

Therefore

$$w_i(n+1) = w_i(n) 2w_i(n)g_i(n)$$
 (22)

A variable step size LMS algorithm where the step size adjustment is controlled by the square of the prediction error. The motivation is that a large prediction error will cause the step size to increase to provide faster tracking while a small prediction error will result in a decrease in the step size to yield smaller misadjustment. The adjustment equation is simple to implement, and its form is such that a detailed analysis of the algorithm is possible under the standard independence assumptions commonly made in the literature to simplify the analysis of LMS algorithms.





Fig. 5. BER performance in QAM



Fig. 6. Conventional method vs. LMS approach

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Fig. 7. LMS method Vs distributed approach



Fig. 8. LMS method Vs minimum error metrics approach



Fig. 9. LMS performance plot



Fig. 10. RLS performance plot



Fig. 11 BER performances of RLS over iteration

## V. CONCLUSION

The proposed system performance analyzes of SC-FDMA system over bank of adoptive filters, with adjustable gains to obtain the desired magnitude response, results fast convergence rate. FIR filter design, although requiring a relatively higher order, permits coupling an almost arbitrary magnitude response with a linear phase response. Here we use a linear- recursive RLS filter as a potential alternative, with the magnitude response for hearing-loss compensation and noise attenuation. A detail analysis of LMS over RLS shows that it is possible to reduce the complexity level. Based on that we have proposed adoptive equalizer has been presented in which the appropriate response can be selected over the existing structure which is scalable for larger block sizes as well as higher filter lengths.

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